Dollar Inflation and Sovereign's Debt Currency Denomination

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This version: September 2022

Abstract

A remarkable observation across Emerging Markets (EMs) in the past two decades is the shift in the currency denomination of its external sovereign debt - from foreign currency (FC) to local currency (LC). This paper first documents and then exploits cross-country variation in the real output correlation with US dollar inflation to explain the heterogeneity of EMs' sovereign external debt currency composition. I provide evidence that countries where real income positively correlates with dollar inflation face high and volatile spreads in dollar-denominated debt. In addition, these countries have a higher share of outstanding LC sovereign debt held by foreigners, and for which this share has increased the most since 2009. Next, I furnish a sovereign default model to rationalize these salient data patterns. A positive correlation between the dollar inflation and EM's real income increases the variance of ex-post available resources, increasing the probability of default ex-post, rendering debt more costly, and jeopardizing consumption smoothing. Lastly, I lay out an extension with nominal frictions and LC debt to highlight the sovereign's main trade-offs, shedding light on incentive problems related to Time Inconsistency. I articulate that the simple model is a particular case of the extended model.

Keywords: Lack of Commitment, Sovereign External Borrowing, Debt Currency Composition, Inflation

JEL classification: D52, E31, E44, F41, H63

1 Introduction

The inability to commit to repaying the debt to foreigners renders the external debt denominated in foreign currency debt expensive to most Emerging Markets (EMs) countries. In addition, the political and macroeconomic instabilities, together with institutional underdevelopment before the late 1990s, constrained the ability of these countries to issue debt to foreigners in their own currencies.

From one point of view, the lack of credibility of monetary policy or its servility to fiscal goals turned the official local currency (LC) debt market solely into one oriented to local market players. Without institutional constraints — such as lawful monetary independence — the power and willingness to inflate away debt in LC effectively pushed EMs to rely on foreign currency (FC) debt to smooth consumption, finance their development or frontload consumption from their future. The reliance on FC debt — mostly denominated in US dollars — made countries choose optimally to default on their repayments agreements on several occasions during the 1980s and 1990s.

In the early 2000s, with the implementation of inflation targetting regimes and the associated stability of inflation dynamics prospects, EMs acquired the capacity to borrow from foreigners in their currency. Arslanalp and Tsuda (2014) documented this pattern of the data. Even though this pattern holds for most EMs, much of the cross-country heterogeneity remains to be exploited and explained.

In this paper, I ask if the EMs' heterogeneity regarding their exposure to dollar inflation can rationalize the heterogeneity regarding their sovereign external debt currency composition. I strategize to tackle this question in several steps. First, I provide suggestive empirical evidence that there is substantial dispersion on how each EMs' real output comove with measured US dollar inflation. I show that countries that happen to have their GDP positively related to dollar inflation face higher spreads for dollar-denominated debt. Also, these spreads are more volatile. I show that these countries have a higher share of LC outstanding debt in the hands of foreigners and the ones for which this share has increased the most since 2009.

Second, I unveil an uncomplicated nominal sovereign default model in which the country borrows in dollars and show that the model delivers qualitative results in line with the data. Intuitively, fixing some positive amount of issuing debt, if real output and dollar inflation positively covariate, then the ex-post variability of the cash-in-hand will be higher than if the covariance were negative. Hence, the sovereign will be more likely to default ex-post, and, as a result, the bond price offered to the sovereign will be less favorable.

I discipline the model with the exogenous structure estimated from the data and show that the model indeed corroborates the economic intuition developed. I solve the model numerically to an acyclical baseline and several alternative specifications for the critical covariance that deliver a strong understanding of the operating forces of the model.

Lastly, I lay out a model with nominal frictions and LC-denominated debt held by foreigners. I demonstrated that the simple model previously solved is a particular case of this enlarged model. In addition, this full-blown model nests out with the lack of credibility of the monetary authority to control domestic inflation rendering the inability to borrow in LC. I conclude by pointing out the limitations of my current analysis and exploiting avenues for going forward with this research agenda.

1.1 Literature Review

Since the seminal work of Eaton and Gersovitz (1981), the literature on sovereign default on the foreigner-held debt has been fruitful. The acknowledgment that the unfeasibility of a commitment technology for repaying these debts is central to this literature: it is nearly impossible to enforce repayment under international jurisdiction against a sovereign. The harshest punishment following default — a temporary exclusion from global financial markets — is sometimes too soft, and the debt burden may be too heavy for the sovereigns to keep rolling debt.

Aguiar and Gopinath (2006) and Arellano (2008) develop models in which the government faces a stochastic output stream and borrows from foreign to sort out undesirable consumption volatility. In their setup, the government is relatively more impatient than international lenders and, hence, is a net borrower. Bonds are short-term (one period) and dollar-denominated, and the government lacks commitment to repaying its obligations to foreigners.

In particular, Arellano (2008) successfully delivers a model consistent with several saliencies from the data. The model predicts that defaults are more likely to happen in "bad times" — periods of realizations of low output — and, as a result, bond spreads are sharply countercyclical. In constructing the model, Arellano (2008) argues that a temporary exclusion from financial markets is insufficient to generate the observed joint dynamics of borrowing and sizable default risk-bearing. In getting the default dynamics right, her model delivers only 6% of the total observed debt stock.

The introduction of longer maturity debt provided manners to get the numbers in the right spot, as explored in Hatchondo and Martinez (2009). Even though the long maturity brings the model closer to data and reality, the model becomes harder to solve. The main issue is that only a fraction of debt matures every period, and marking-to-market the non-maturing fraction of debt renders the current bond price dependent on continuation borrowing policies, contingent on not defaulting. This feature leads to several numerical annoyances but, in particular, non-convergence. Chatterjee and Eyigungor (2012) engineers an algorithm that assures convergence for a model that features a long-term defaultable bond, even though the algorithm might face numerical instability under some parametrizations and is hard to generalize to more detailed environments, such as in Gordon and Guerron-Quintana (2018).

More recently, Dvorkin et al. (2021) built on the extensive literature on discrete dynamic models by adding taste shocks to an extended model of sovereign default. The policy functions become probability distributions over model-specific perturbations that co-move slowly with the sequence of price schedule updates, rendering convergence to the "benchmark" algorithm.

Another desirable feature of their algorithm is that policy and value functions can be rewritten in closed form, speeding up coding and computing time. But there is no such thing as free lunch, and the solution is subject to numerical instabilities and heavily dependent on user-specified grids. Because of distributional assumptions on the taste shocks, adding more grid points to the debt space tilts down the probability of default computed by the model. Despite these limitations, the approach is quickly becoming the gold standard for dealing with longer-maturity defaultable bonds. Gordon (2019) develops additional tools to exploit properties of the policy functions to improve efficiency.

Departing from the baseline model drafted by Arellano (2008), the literature has exploited the role of safe short-term savings technology quantitatively in the settings of sovereign default, usually referred to as international reserves. Alfaro and Kanczuk (2009) pioneers this branch of this literature, relying on settings with short-term bonds. The main lesson is that the short-term liability charging a higher return than the one accrued from the short-term international reserves accumulation turns into an optimal behavior that does not accumulate assets.

The introduction of long-term debt creates a maturity wedge between liabilities and assets. Because borrowing costs fluctuate over the business cycles for a given amount of debt chosen, the government can optimally choose to transfer resources from high-borrowing prices states to low-borrowing cost states using international reserves, as explored by Bianchi et al. (2018) and Bianchi and Sosa-Padilla (2020).

Another avenue developed by the literature, more closely related to this project, is greatly influenced by Schmitt-Grohé and Uribe (2016). The main departure from standard literature is to work with an economy subject to downward nominal wage rigidity. In such a setting, a low realization of output triggers a downturn in the non-tradable section that calls for a decrease in real wages. The two potential solutions are a cut in nominal wage denominated in domestic currency or a nominal exchange rate depreciation, in line with the contribution of Friedman et al. (1953). Mixing a fixed exchange rate regime and the downward nominal wage rigidity translates into real friction, causing a demand externality channel and involuntary unemployment in periods of low output.

Building on this nominal model, Na et al. (2018) develops a decentralized economy to rationalize a salient feature of the data known as "Twin D's". They document that sovereign defaults often combine with excess nominal exchange rate depreciation. In their model, because of the embedded demand externality, a sequence of low realization of output drives up the pressure to default and devalue the domestic currency.

Bianchi and Sosa-Padilla (2020) develop a model with international reserves to exploit the macroeconomic stabilization role of international reserves. They show that economies that face nominal wage downward rigidity and adopt a fixed exchange rate regime benefit from hoarding global assets. The insight is that the government can sell reserves ex-post to exploit a demand externality steaming from the tradable sector. Therefore, having funds today lowers the average and dispersion of the unemployment rate ex-post, provided that the government's portfolio bears some default risk.

Bianchi and Mondragon (2022) is another milestone paper exploiting a nominal environment in sovereign default models. In their environment, there is roll-over risk in that spirit of Cole and Kehoe (2000), debt is long-term, and nominal wage is downward rigid. Their main result is that an economy that gave up on monetary policy independence becomes more prone to a self-fulfilling crisis, whenever their creditor refuses to lend in expectation of subsequent default, the government finds optimal to default in the first place.

Arslanalp and Tsuda (2014) inaugurated a new dataset to tracks the ownership of sovereign debt outstanding in many EMs - extended more recently to include Low-Income Countries. The data shows a sustained increase in the share of LC debt issued by EMs that is owed to foreigners, even though the foreign currency is the dominant currency for official external debt denomination.

Du and Schreger (2016) exploits a comprehensive dataset on bond prices from several EM to show the existence of a premium for LC debt. Du and Schreger (2022) further connects corporate sector debt currency denomination and sovereign default risk. A higher reliance of the corporate sector on foreign currency debt is usually associated with more salient sovereign default risk. The authors study the trade-offs between inflation and default in an environment equipped with multi-currency debt portfolios.

Ottonello and Perez (2019) shows that EM's sovereign external debt composition tends to be pro-cyclical. The authors introduce a model with costly inflation denominated in LC in which the government can borrow from abroad using LC and FC debt instruments. The introduction of LC debt renders the local inflation interconnected with the foreign bondholders' pricing scheme since the nominal devaluation relates directly to the ex-post inflation.

Low realizations of outputs make debt repayment more costly and are usually associated with a softer debt burden because the exchange rate movement makes the real payments less expensive ex-post. The authors show that, on the other hand, if the nominal exchange rate is exogenously countercyclical, the debt denominated in local currency provides natural hedging for the government. Of course, the international lenders foresee these dynamics ex-ante, and the borrowing prices adjust accordingly.

Ottonello and Perez (2019) is the paper most related to my project. Several distinctions are in place. First, I document a data pattern disregarded by the authors. They argued that dollar inflation is more negligible on average and less volatile than local currency inflation in EMs. While this is true, it is more plausible to think about the inflation in dollars to be "more" exogenous than the nominal exchange rate dynamics. The authors instead choose in the opposite direction.

A significant difference is that while Ottonello and Perez (2019) works mainly on a default-free environment, I emphasize the sturdy commitment issues arising from the lack of enforcement technologies. I argue that depending on the covariance structure between the local output and the dollar inflation, EMs might find it optimal to tilt their external debt towards their LC. Lastly, while Ottonello and Perez (2019) tries to target an "median" EM in their numerical exercises, I look over the cross-country heterogeneity in this key covariance metric mapping into the cross-country heterogeneity in the external sovereign debt currency composition.

2 Empirical Support

In this section, I provide suggestive evidence supporting the link between exposure to US dollar inflation and the sovereign external debt currency composition for a set of EMs. Because this project focuses on explaining cross-country variation in external debt, I restrict attention to EMs for which high-quality, comparable high-frequency output data is available. Below I provide a succinct overview of the data and include further details in the Appendix.

I rely on OECD's dataset for Quarterly national accounts. In this data set, I find data for 15 EMs that will be my universe of analysis.[1] I collect real GDP measured in 2015 US dollar units for all these countries in addition to the GDP Price Deflator series for the US, which I use to compute measured US dollar inflation. I restrict my sample from 1997Q1 to 2019Q4 to incorporate as much data as possible and avoid the COVID-19 pandemic period.

In dealing with these series, I follow a similar approach to the one in Hur et al. (2021). First, as it is standard in sovereign default and business cycle literature, I extract from the data a "trend" and a "cycle" components.[2] In logs, the sum to the actual or realized series. For the dollar inflation metrics, I first take differences on the log-extracted equation using the GDP Price Deflator to compute a proxy for the trend inflation and what I label cyclical inflation. The equations below highlight the approach:

[1]These countries are from Latin America (Argentina, Brazil, Chile, Colombia, Costa Rica, and Mexico), Europe (Bulgaria, Hungary, Poland, Romania, Russia, and Turkey), Asia (Indonesia and India), and Africa (South Africa).

[2]I use the two-sided HP filter with smoothing parameter $\lambda = 1600$ to extract the "trend" and "cycle" components. The data is posted on a quarterly frequency.

$$\log(y_{i,t}) = \log(y_{i,t}^{trend}) + \log(y_{i,t}^{cycle}) \tag{1}$$

where $y_{i,t}$ is the measured real output for country i at date t.

$$\log(P_t) = \log(P_t^{trend}) + \log(P_t^{cycle}) \tag{2}$$

where P_t is the GDP Price Deflator Index for the US measured in dollars at date t. Applying the first difference to equation 2:

$$\begin{aligned} d\log(P_t) &= d\log(P_t^{trend}) + d\log(P_t^{cycle}) \\ \pi_t &= \pi_t^{trend} + \pi_t^{cycle} \end{aligned} \tag{3}$$

From equation 3, I constructed a proxy of the trend inflation and the cyclical inflation in dollars. Two properties are worth emphasizing. First, this specification allows for medium-run movements in inflation captured by the trend inflation component. Second, this formulation makes trend and cyclical inflation multiplicative, which is a desirable property for modeling: only the cyclical part of inflation matters.

Figure 1 portrays two examples: Mexico and Colombia. On the one hand, there is significant covariance between the cyclical component of the output of Mexico and the cyclical component of US dollar inflation. On the other hand, there is virtually no relationship between the two series in the case of Colombia.[3]

In what follows, I simplify notation by dropping the "cycle" superscript. All the remaining analysis focus on these series. I take into account that both cyclical inflation and output are persistent. To handle this, I cast the two series into VAR(1) specification and focus on the

[3] The Appendix displays the same scatter plot for all countries I consider in this analysis.

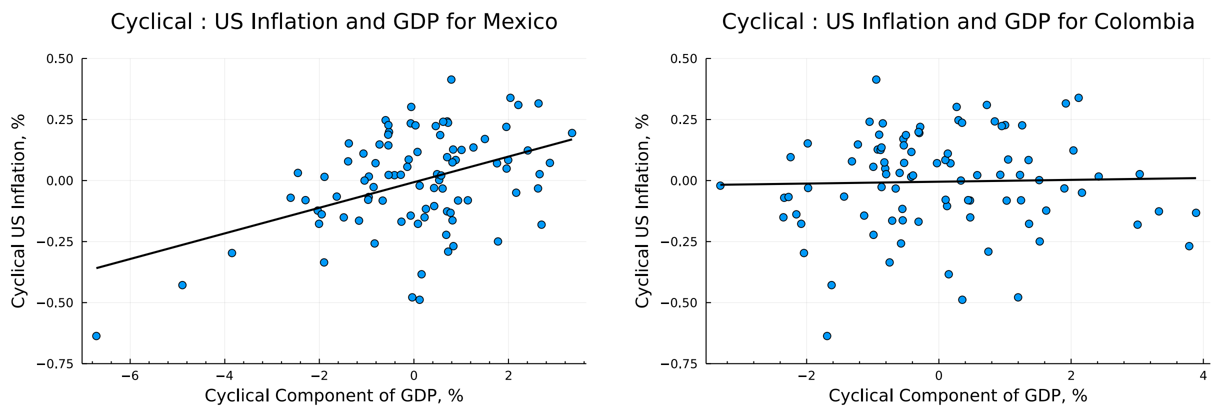

Figure 1: Cyclical US Inflation and GDP for Mexico (left) and Colombia (right).

covariance of the innovations to both series, as follows:

$$\begin{bmatrix} \log(y_{i,t+1}) \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho^i_{yy} & \rho^i_{\pi y} \\ \rho^i_{y\pi} & \rho^i_{\pi\pi} \end{bmatrix} \begin{bmatrix} \log(y_{i,t}) \\ \pi_t \end{bmatrix} + \begin{bmatrix} \epsilon^i_{yt+1} \\ \epsilon^i_{\pi t+1} \end{bmatrix} \tag{4}$$

or

$$s' = A^i s + \epsilon'$$

with

$$s \equiv \begin{bmatrix} \log(y_{i,t}) \\ \pi_t \end{bmatrix} \quad \text{and} \quad A^i \equiv \begin{bmatrix} \rho^i_{yy} & \rho^i_{\pi y} \\ \rho^i_{y\pi} & \rho^i_{\pi\pi} \end{bmatrix}$$

where the innovations are

$$\begin{bmatrix} \epsilon^i_y \\ \epsilon^i_\pi \end{bmatrix} \sim \mathbf{N}(\vec{0}, \Sigma_i) \quad \text{with} \quad \Sigma_i \equiv \begin{bmatrix} \sigma^2_{yi} & \sigma_{\pi yi} \\ \sigma_{yi\pi} & \sigma^2_{\pi} \end{bmatrix} \tag{5}$$

For each country i in my sample, I compute the four memories coefficients, located in A^i, and the three distinct elements from the variance-covariance matrix, Σ_i. The estimated values are organized in Table 1.

A few comments are in place. The second column suggests that output is highly persistent for the countries in the sample. As the fifth column makes evident, the inflation cycle is

Country	ρ_{yy}	$\rho_{\pi y}$	$\rho_{y\pi}$	$\rho_{\pi\pi}$	σ_y	$\sigma_{y\pi}$	σ_π
Argentina	0.86	0.48	0.01	0.11	1.88	0.09	0.19
Brazil	0.78	0.47	0.02	0.16	1.02	0.05	0.20
Chile	0.82	0.41	0.02	0.17	1.01	0.04	0.20
Colombia	0.82	0.45	-0.01	0.21	0.83	0.02	0.20
Costa Rica	0.78	0.13	0.01	0.20	0.84	0.02	0.20
Mexico	0.78	0.95	0.02	0.13	0.91	0.07	0.20
Bulgaria	0.59	0.72	-0.01	0.20	1.87	0.00	0.20
Hungary	0.82	0.81	0.02	0.18	0.76	0.02	0.20
Poland	0.69	0.12	0.00	0.20	0.86	0.03	0.20
Romania	0.86	0.13	-0.01	0.20	1.18	0.05	0.20
Russia	0.84	1.34	0.00	0.20	1.33	0.08	0.20
Turkey	0.81	0.18	0.02	0.08	1.97	0.07	0.19
South Africa	0.86	0.73	0.01	0.20	0.45	0.02	0.20
India	0.80	-0.22	0.05	0.10	0.88	0.00	0.19
Indonesia	0.76	0.62	0.00	0.21	1.34	0.03	0.20
mean	0.79	0.49	0.01	0.17	1.14	0.04	0.20
minimum	0.59	-0.22	-0.01	0.08	0.45	0.00	0.19
median	0.81	0.47	0.01	0.20	1.01	0.03	0.20
maximum	0.86	1.34	0.05	0.21	1.97	0.09	0.20
standard deviation	0.07	0.38	0.02	0.04	0.44	0.03	0.00

Table 1: Parameters estimated from the VAR approach for each of the 15 countries in the samples and some descriptive statistics.

estimated to be somewhat persistent but much less than output. From the sixth and eighth columns, we learn that the estimated standard deviation on innovations to output is between five and nine times higher than the counterpart for inflation.

The most relevant ingredient from the table is the numbers from the seventh column. This column shows that the estimated covariance of innovations on output and inflation is dispersed. They range from 0.00 (rounding from a slightly negative number) and 0.09. The mean of the estimates is 0.04, and the median is 0.03. Later, I use these numbers to discipline the model developed in the next section.

Next, I work with data for the spreads on US dollar-denominated debt. For this, I use the EMBI+ for the same set of EMs. EMBI+ stands for the Emerging Market Bond Index Plus and tracks the total return of bonds denominated in USD issued by EMs. I collected the data from the Global Economic Monitor, published by the World Bank. [4] Assets must meet strict liquidity requirements in secondary markets and are Brady bonds, loans, or Eurobonds. Here I focus on the G series, which only considers sovereign debt.

Country	Mean	Std	Median	Max	Min
Argentina	1486	1753	719	6847	203
Brazil	462	362	290	2057	143
Chile	149	54	142	383	55
Colombia	326	196	230	986	108
Costa Rica	431	59	422	582	339
Mexico	271	134	236	944	98
Bulgaria	342	276	240	1366	43
Hungary	173	138	124	650	14
Poland	139	80	120	345	25
Romania	...	...	...	...	...
Russia	604	1002	257	5919	96
Turkey	398	202	320	1048	162
South Africa	250	113	250	669	58
India	144	19	147	182	99
Indonesia	241	137	229	891	0

Table 2: Summary statistics for EMBI+ Series G, on spreads. Spreads are measured in basis points denominated in USD and relative to a 5y Treasury bond issued by the US government. Data is not available for Romania. For Bulgaria, the data ended in December 2013. Data for Costa Rica and India start in February 2015 and in April 2004 for Indonesia. For the other countries, the data start in December 1997, except for Chile, which starts in May 1995. The zero in the minimum column for Indonesia is odd and took place in the first four months of 2013, at the beginning of the series. Here “Std” stands for “standard deviation”, “Max” stands for “maximum” and “Min” stands for “minimum”.

Figure 2 shows two scatter plots. The measured covariance between the cyclical component of US dollar inflation and the cyclical component of the output for the set of EMs is exhibited in the horizontal axis. In the vertical axis, there are two different variables. The plot on the left shows the time series average (mean) spread in bonds denominated in USD, as measured by the EMBI+. The plot on the right reveals that same relation but with the spread’s standard deviation (or volatility).

[4] data is not available for Romania. For Bulgaria, the data ended in December 2013. Data for Costa Rica and India start in February 2015 and in April 2004 for Indonesia. For the other countries, the data begin in December 1997, except for Chile, which starts in May 1995.

The figure conveys that a higher covariance between real, local output, and USD inflation (in cycles) is associated with higher spreads. In addition, these spreads are also more volatile.

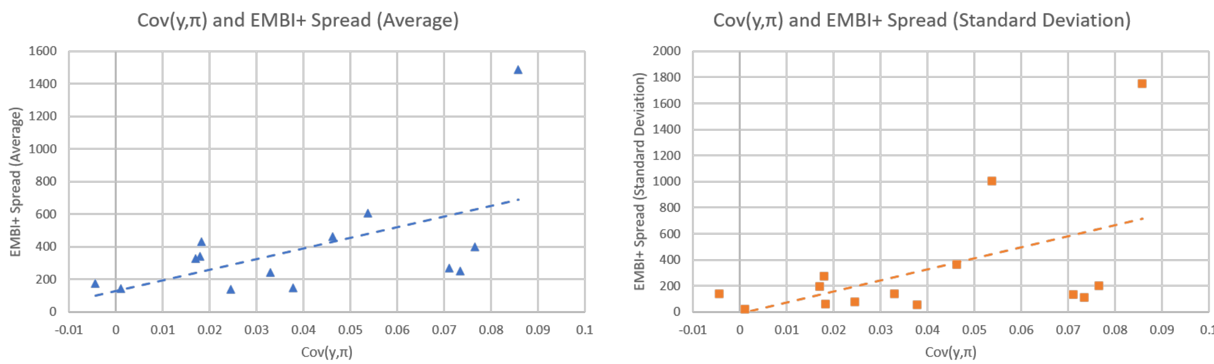

Figure 2: Measured covariance between real, local output and US dollar inflation, both in cycles as defined above. The percent change in GDP Deflator measures US dollar inflation. An observation — triangle at the left or square at the right — corresponds to a country in the universe.

Next, I show that EMs have tilted their sovereign liabilities owed to foreigners away from foreign currency (primarily dollars) towards their local currency. For that, I built on the data of Arslanalp and Tsuda (2014), extended for 2022. The data is posted quarterly, and the last data point is 2019Q4. Figure 3 shows countries where I do not work on quarterly GDP data due to availability and comparability issues. The goal of adding additional countries is to emphasize the widespread trend of borrowing in LC among EM and, importantly, the amount of cross-country heterogeneity.

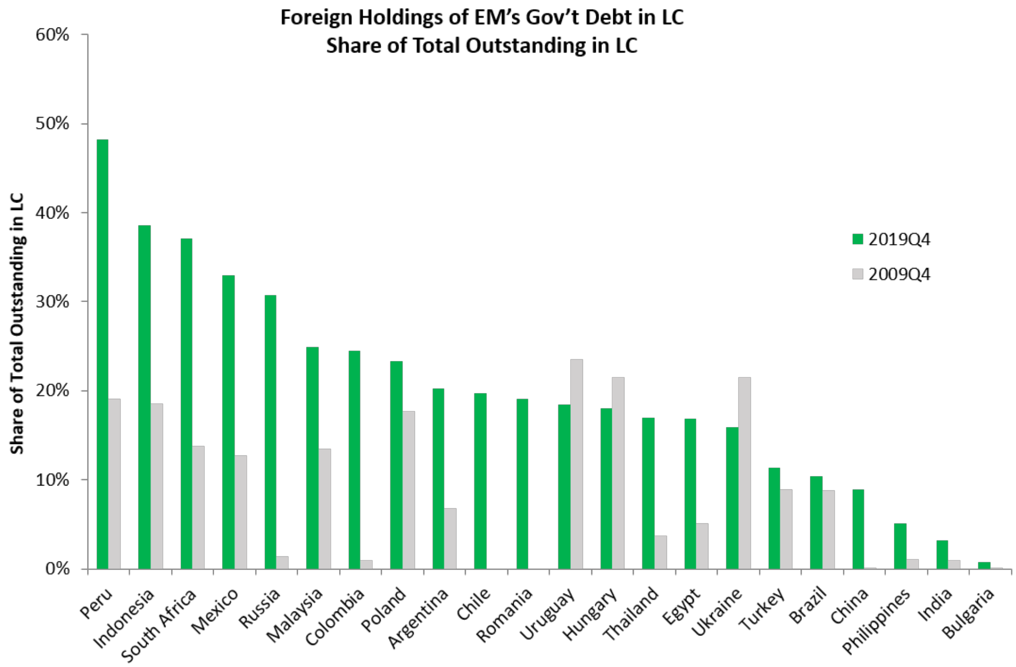

Figure 3: The figure shows the share of sovereign liability denominated in LC that is owed to foreigners. Data from Arslanalp and Tsuda (2014), extended to 2022. Data is not available for Costa Rica. Data is not available for Chile and Romania for 2009Q4 but is available for 2019Q4. These countries are in the mid portion of the horizontal axis.

Figure 4 exhibits two scatter plots. As in figure 2, the horizontal axis shows the measured covariance between the cyclical component of dollar inflation and the cyclical component of the output for the set of EMs. The left panel shows the share of LC debt owed to foreigners in 2019Q4, while the right panel change in p.p. from this share from 2009Q4 to 2019Q4. The figure emphasizes that countries with higher measured covariance on average not only exhibited a higher share of their LC debt owed to foreigners in late 2019 but also had a higher increase in this share from late 2009.

This project is about exploiting how the cross-country heterogeneity into the measured covariance of interest maps into the dynamics of EMs's liability currency composition during the two decades. Even though the evidence provided from figures 1, 2, 3, 4 should be taken with caution. The data suggest that a higher covariance between local output and dollar inflation is usually associated with higher borrowing costs — measured by higher and more

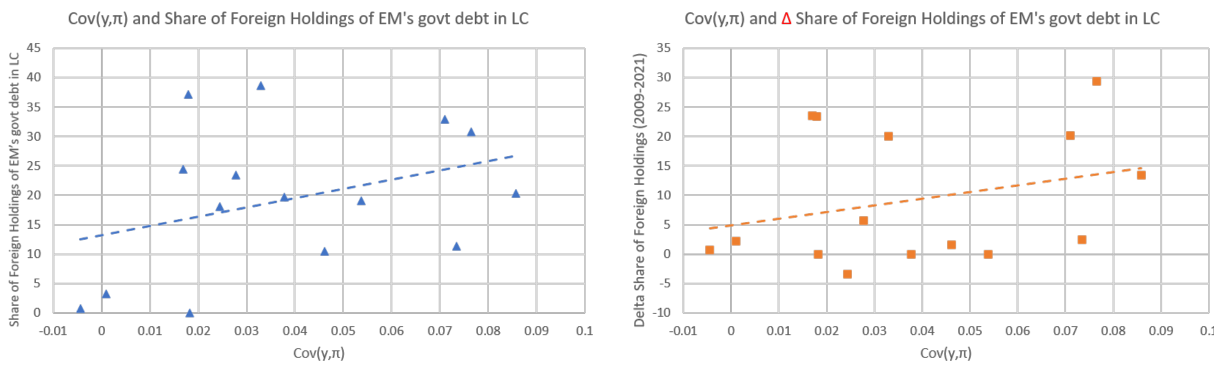

Figure 4: The figure relates the share of sovereign liability denominated in LC owed to foreigners and the measured covariance between the cyclical components of EMs real, local output, and the US dollar inflation. The dashed line in each picture shows the best-fit in a linear regression with a constant.

volatile spreads — and associated with a higher share of debt denominated in LC owed to foreigners.

Going forward, I will develop a model to rationalize these salient patterns of the data. Specifically, I work with a model that shows that dollar debt is more expensive in equilibrium when the covariance between the dollar and real, local output is positive than when this covariance is zero. When the covariance is negative, dollar inflation works partly as insurance, and dollar debt is cheaper.

3 The Model

The environment builds on the traditional sovereign default models, such as Eaton and Gersovitz (1981), Arellano (2008), Aguiar and Gopinath (2006), with elements from Ottonello and Perez (2019). I consider a small open economy (SOE) in discrete time and infinite horizon.

There is a single final freely-traded good in the economy, endowed to representative households each period. The representative household does not have access to international financial markets, but the government does. Government and households share the same utility function and discount factor. As it is common in the literature of sovereign external borrowing, I assume that the government is endowed with sufficiently broad fiscal instruments to choose the allocations directly — consumption, and borrowing — on behalf of the households.

The government can issue nominal bonds in international financial markets. The bonds are short-term, one-period denominated in US dollars, implying that the US dollar denominates the relevant budget constraint for the government.[5] The government lacks the commitment to repay these bonds in the following periods when they are due. Hence, bonds are not default-free, and lenders price the default risk from an ex-ante perspective.

The government faces two costs from default. First, there is a random period of exclusion from financial markets. Second, bad credit standing reduces local, real output. Default wipes away the entire outstanding stock of external liability and can be used to smooth consumption. Whenever the government reenters the global financial markets, it starts over with zero debt outstanding.

I assume that the lenders are risk-neutral with deep pockets and equipped with rational expectations. In the environment, there is no information friction. There is common knowledge about the EM real output realizations and inflation, in addition to the joint conditional distribution prospects at any given state and time.

3.1 Preferences and Decisions

Since the government and the household share the same utility function, I emphasize the role of the government in what is next. I start describing the preferences.

The government values the consumption of a single final good, and its ultimate goal is to maximize discounted expected utility. Flow utility depends on consumption, as Inada conditions are satisfied for the utility function u:

$$\forall c \geqslant 0: \quad u'(c) > 0, \quad u''(c) < 0, \quad \text{and} \quad \lim_{c \to 0^+} u'(c) = +\infty \tag{6}$$

[5] An extension with local currency is provided later.

The government has a discount factor β, assumed to be strictly lower than the inverse of the rate at which international lenders discount real flows.

The government can use bonds denominated in US dollars either smooth out income fluctuations or to frontload consumption, as emphasized in Alfaro and Kanczuk (2009). The nominal value of these bonds are denoted by B_t, while the real — or de-trended value — is $b_t \equiv \frac{B_t}{P_{t-1}}$, where P_{t-1} is the price level at period $t-1$. Since the borrowing is in US dollars and there are no other frictions, the appropriate price level is denominated in US dollars too. The nominal price of these bonds is q_t and reflects that the government has the option to default ex-post.

Under repayment, the government can choose how much to consume and borrow for the next period. Under default, the government consumes the penalized output and cannot borrow from abroad.

3.2 Endowment and Exogenous Uncertainty

At each date t, the representative household is endowed with a stochastic amount of real output, y_t. I assume that trading this good with international markets is frictionless and that the realization of output is potentially related to the realization of US dollar inflation. Since the trade of this good is frictionless, the Law of One Price holds, and external dollar inflation is the same as internal dollar inflation.

Let the inflation in dollar be denoted by π_t, where $1 + \pi_t = \frac{P_t}{P_{t-1}}$. The joint dynamics of exogenous local real output and (global) US dollar inflation is given by the following system:

$$\begin{bmatrix} \log(y_t) \\ \pi_t \end{bmatrix} = \begin{bmatrix} \rho_{yy} & \rho_{y\pi} \\ \rho_{\pi y} & \rho_{\pi\pi} \end{bmatrix} \begin{bmatrix} \log(y_{t-1}) \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{yt} \\ \epsilon_{\pi t} \end{bmatrix} \tag{7}$$

with

$$\begin{bmatrix} \epsilon_{yt} \\ \epsilon_{\pi t} \end{bmatrix} \sim \mathbf{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_y^2 & \sigma_{y\pi} \\ \sigma_{\pi y} & \sigma_\pi^2 \end{bmatrix} \right) \tag{8}$$

The approach I follow here is similar to Hur et al. (2021): I analyze a model in which local output y and dollar inflation π systematically correlate, but I do not take a particular stand on why this co-movement happens in the first place. The main idea is that international investor and EM's sovereign would take as given such correlation when formulating their optimal policies.

There is also exogenous uncertainty regarding the length of financial autarky — the stage at which the government cannot borrow from international markets. Under default, the government collects an output $y - h(y)$ where $h(y) \geqslant 0$ is an weakly increasing function of the endowment level y. I emphasize that h is deterministic, and joint dynamics of output do not change during default. If the government defaults at some date t, it can regain access to financial markets with probability $\theta \in (0, 1)$, independent of output realizations, and i.i.d. over states and time.

3.3 Budget Constraints

Under good credit standing, the government faces the following budget constraint:

$$P_t c_t + B_t \leqslant P_t y_t + q_t B_{t+1} \tag{9}$$

where the government chooses at period t the amounts c_t and B_{t+1} taking as given P_t and y_t. The nominal bond price q_t depends on B_{t+1} as higher debt for tomorrow might be associated with a higher probability of default ex-post. Normalizing the budget constraint by the appropriate price level — choices made at t divided by P_t —, the relevant budget constraint under repayment is given y

$$c_t + \frac{b_t}{1 + \pi_t} \leqslant y_t + q_t b_{t+1} \tag{10}$$

so that higher-than-expected US dollar inflation alleviates the debt repayment burden and tends to make it more likely for the government to repay its debt. Under default, the budget constraint is

$$P_t c_t \leqslant P_t(y_t - h(y_t)) \tag{11}$$

So that we can rewrite it by dividing out by the price level to write

$$c_t \leqslant y_t - h(y_t) \tag{12}$$

3.4 International Lenders

I assume that international lenders are risk-neutral with deep pockets. They discount future real flows by $R = 1 + r$, assumed to be time-invariant for simplicity.[6] These lenders have rational expectations and price default and inflation dilution risk perfectly. The nominal price of a risk-free bond is given by

$$q_t^{RF} = \frac{1}{R}\mathbb{E}_t\left[\frac{p_t}{p_{t+1}}\right] = \frac{1}{R}\mathbb{E}_t\left[\frac{1}{1+\pi_{t+1}}\right] \tag{13}$$

In what follows, we let d_t be the indicator function that tracks the credit standings of the government:

$$d_{t+1} = \begin{cases} 0 & \text{if the government is in good credit standing at date t+1} \\ 1 & \text{if the government is in bad credit standing at date t+1} \end{cases} \tag{14}$$

Then, the price of debt is given by

$$q_t = \frac{1}{R}\mathbb{E}_t\left[\frac{p_t}{p_{t+1}}(1-d_{t+1})\right] = \frac{1}{R}\mathbb{E}_t\left[\frac{1}{1+\pi_{t+1}}(1-d_{t+1})\right] \tag{15}$$

The price of debt is zero if $d_{t+1} = 1$ for every possible state tomorrow, that is, if, regardless of the state tomorrow, the government would find it optimal to default. If the government

[6] As emphasized by Taylor (1993), the nominal risk-free rate tends to be adjusted by the nominal authorities following a spike in inflation. I do not take it into account in my analysis for simplicity, but I suspect that the qualitative assessment would not change.

would find it optimal to default in some states but not all of them, then it follows that the price of a bond is strictly positive.

3.5 Recursive Formulation

Next, I cast this economy in a planning problem in the recursive form. I work directly with the de-trended system, normalized by the price level. There are three relevant state variables, two exogenous—y and π, real, local income and US dollar inflation—and one endogenous—b, the debt level normalized by the appropriate price level. Let $s \equiv (y, \pi)$ be the aggregate exogenous state. The value of the government is given by

$$V(b,s) = \max_{d \in \{0,1\}} \left\{ (1-d)V^r(b,s) + dV^d(s) \right\} \tag{16}$$

where V^r represents the value of repayment and V^d represents the default value. Notice that the default value is independent of debt since, under bad credit standing, the debt must be equal to 0.

The value of default is given by

$$V^d(s) = \max_{c} \left\{ u(c) + \beta \mathbb{E}_{s'|s} \left[\theta V(0,s') + (1-\theta) V^d(s') \right] \right\} \tag{17}$$

$$\text{subject to}$$

$$c \leqslant y - h(y)$$

Under default, there is no choice to be made as penalized output pins down the current consumption. The discounted continuation value has two components. With probability θ, the government regains access to financial markets in the following period, and with probability $1-\theta$, the government continues in financial autarky. If the government can borrow and save again next period, it returns with 0 debt.

The value of repayment is given by

$$V^r(b,s) = \max_{c,b'} \left\{ u(c) + \beta \mathbb{E}_{s'|s} \left[V(b',s') \right] \right\} \tag{18}$$

$$\text{subject to}$$

$$c + \frac{b}{1+\pi} \leqslant y + q(b',s)b'$$

Under repayment, the government repays its debt and chooses between consumption c and borrowing for next-period b'. The continuation is the value of the government at state (b',s'), which is the maximum between the repayment and default value at this given state.

It is essential to realize that the bond's price depends on both the amount borrowed b' and the aggregate exogenous state s. All else equal, borrowing more today translates into tighter budget constraint under repayment and can increase the default probability but not decrease it. In addition, the current inflation dynamics are informative about the next-period inflation if there is at least some persistence in the process specified in 7. For the same reason, the current endowment is informative about the next-period endowment and, therefore, the next-period default probabilities.

The government defaults with state (b,s) if the value under default is strictly higher than the value under repayment, that is $V^d(s) > V^r(b,s)$. Hence, the default decision is summarized as follows:

$$d(b,s) = \begin{cases} 0 & V^d(s) \leqslant V^r(b,s) \\ 1 & V^d(s) > V^r(b,s) \end{cases} \tag{19}$$

With this definition on hand, the price schedule is

$$q(b',s) = \frac{1}{R} \mathbb{E}_{s'|s} \left[\frac{1}{1+\pi(s')} (1 - d(b',s')) \right] \tag{20}$$

where $\pi(s')$ is the inflation rate associated with state s'. Given this price, I define one auxiliary object of interest, the annualized spread:

$$spread(b', s) \equiv \left(\frac{1}{q(b', s)}\right)^4 - \left(\frac{1 + r}{\mathbb{E}_{s'|s}\left[\frac{1}{1+\pi(s')}\right]}\right)^4 \tag{21}$$

I now use these pieces to define the recursive equilibrium below.

Definition 3.1 (Recursive Equilibrium) The Recursive Equilibrium is a set of

a) Value functions $\left\{V(b,s), V^d(s), V^r(b,s)\right\}$;

b) Associated policy functions for $\left\{d(b,s), c(b,s), b'(b,s)\right\}$; and

c) a Price Function $q(b', s)$

such that

I) Taking as given the bond price, the policies solve the government's problem

II) The bond price is consistent with break-even for the international lenders

3.6 Economic Intuition

Before moving into calibration and numerical exercises, I discuss the economic intuition of the simplified version of the model. I start considering three special cases for the stochastic environment. The first one, which I refer to as baseline, is where the real output in the EM does not correlate with the US dollar inflation. Then, I explore one case in which this correlation is positive and another negative.

First, rewrite the repayment under default as follows, denoting the cash-in-hand as φ:

$$c - q(b', s) \leqslant y - \frac{b}{1 + \pi} \equiv \varphi \tag{22}$$

Now, provided that $b > 0$, it is straightforward that

$$\frac{\partial \varphi}{\partial y} > 0 \qquad \text{and} \qquad \frac{\partial \varphi}{\partial \pi} > 0 \tag{23}$$

So higher endowment and higher inflation increase the cash-in-hand, making it more likely to repay debt. While higher endowment means higher default costs, higher inflation partially dilutes debt, and defaulting gives up this benefit.

In the baseline case — $cov(y, \pi) = 0$—, the inflation variability is an additional source of volatility to the cash-in-hand, other than the one coming from the income fluctuations. If utility is concave, then this extra volatility is undesirable. Since markets are incomplete and the bond is not state-contingent, the government would use the default option to smooth out consumption if unfavorable output-inflation states realizes.

The extra variability in cash-in-hand tends to increase the likelihood of default for a given $b > 0$. The reason is that "good" shocks do not increase the likelihood of repayment if the repayment was happening with certainty in some states, but it could potentially enlarge the set of conditions for which default is desirable for the government.

Since the lenders recognize this behavior from an ex-ante perspective, the price schedule consistent with the default policy would adjust so that the same positive debt would imply a higher spread relative to the case of no inflation volatility.

In the first notable case, the covariance between output and US dollar inflation is positive — $cov(y, \pi) > 0$. In this case, periods of high output tend to happen in junction with periods of high US dollar inflation, but the same holds for low output and low inflation. Because of equation 23 and the mechanism explained earlier, for a given level of debt $b > 0$, the volatility of cash-in-hand will be higher relative to the baseline case.

Hence, from an-exte perspective, lenders will correctly assign a higher probability of default ex-post, and the government will face worse borrowing conditions: spreads to the risk-free rate will be higher.

In the second particular case, the covariance of interest is negative — $cov(y, \pi) < 0$. This case implies that periods of low output that make default more tempting ceteris paribus are usually associated with the period of high inflation, which in turn dilutes part of the debt due. Therefore, US dollar debt is partial, incomplete insurance against income fluctuations.

This feature of the debt denominated in dollars renders spreads lower for a given amount of borrowing relative to the baseline. The economic intuition suggests that this case would allow the government to carry less risk in equilibrium, borrow more, and spread to be lower, possibly less volatile. The case in which the covariance is positive would mirror this one: less borrowing, more risk, and higher spreads. The baseline case would fit in between the two alternatives.

4 Calibration and Numerical Illustration

In this section, I describe the parametrization, numerical methods, and simulations I carry out with the simplified model. One period is a quarter.

I start noticing that, as it is long recognized, the canonical model of sovereign default with short-term bonds can not generate simultaneously spreads moments (mean and volatility) and indebtedness (mean debt-to-output ratio) consistent with the data. This result was illustrated in Aguiar and Gopinath (2006) and Arellano (2008) and later formalized by Cao and Gordon (2019).

Since I work with short-term debt, I emphasize the amount of borrowing and sacrifice the moments on spreads. The reason is that the dilution through the inflation channel, an essential part of the argument, is more relevant whenever debt-to-output is high. Nevertheless, in order to achieve plausible figures at this moment, the debt must be nearly risk-free. Hence, I take the results below as mere illustrations of the main channel.

4.1 Functional Forms and Calibration

For the utility function and default cost, I borrowed the function forms from Chatterjee and Eyigungor (2012), which arguably became the mainstream reference for the literature. For the utility function, I use the CRRA function:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

and

$$h(y) = \max\left\{0, d_1 y + d_2 y^2\right\}$$

This function allows for two parameters d_1, d_2 to target two data moments: the mean and volatility of spreads, as explored in Chatterjee and Eyigungor (2012) and Bianchi et al. (2018).

There are two blocks of parameters. The first block consists of six parameters: (i) σ describing the risk aversion coefficient; (ii) θ the probability of regaining access to financial markets after a period of exclusion following a default episode; (iii) r, the net rate at which the international lenders discount real flows; (iv) β, describing the discount factor of the government; (v) d_1, a linear term controlling the default costs; (vi), d_2, a quadratic term controlling the real costs.

The second block consists of the seven additional parameters specifying the dynamics of the exogenous states, namely A and Σ in the counterpart of equation 4. In the matrix A, there are four memories components: ρ_{yy}, $\rho_{y\pi}$, $\rho_{\pi y}$, $\rho_{\pi\pi}$. The first one describes the persistence of output, while the fourth describes the persistence of inflation. The second and third specify the cross effect from output to inflation and the converse. In turn, there are three other unique parameters: σ_y^2, $\sigma_{y\pi} = \sigma_{\pi y}$ and σ_π^2. At the core of the mechanism is $\sigma_{y\pi} = \sigma_{\pi y}$.

Provided the numerical results are meant to be illustrative, I fix parameters (σ, θ, r) as in Chatterjee and Eyigungor (2012). I try to match a target to the debt-to-output ratio using

β and pick illustrative numbers for d_1, d_2. In addition, I use ρ_{yy} and $\rho_{\pi\pi}$ as the median estimates from the VAR estimates in table 1, and constrain $\rho_{y\pi}$ and $\rho_{\pi y}$ to be zero.[7] I also use the median of the estimates for σ_y, σ_π from table 1. Again, I constrain $\sigma_{y\pi}$ to be zero as a baseline specification. Then, I take the maximum absolute value —0.0858×10^{-4}— for this parameter to be either the positive or negative covariance of output and inflation. Table 3 shows the parameters values.

Parameter	Value	Explanation or Target
σ	2.0000	Standard, Chatterjee and Eyigungor (2012)
θ	0.0385	Expected exclusion 6.5 years, Chatterjee and Eyigungor (2012)
r	0.0100	Annual real rate $\approx$ 4%, Chatterjee and Eyigungor (2012)
β	0.8750	Targeted $b/y = 64.14\%$
d_1	-0.5086	Illustrative
d_2	0.5436	Illustrative
ρ_{yy}	0.8091	Var estimate, median.
$\rho_{y\pi}$	0.0000	Illustrative
$\rho_{\pi y}$	0.0000	Illustrative
$\rho_{\pi\pi}$	0.1971	Var estimate, median.
σ_y	1.0063×10^{-2}	VAR median estimate.
$\sigma_{y\pi}$	0.0000	Acyclical baseline, $\pm 0.0858 \times 10^{-4}$
σ_π	0.1964×10^{-2}	VAR median estimate.

Table 3: Parametrization for the numerical exercises.

Concretely, I choose β to meet a target on the external debt as a share of output. For that, I used World Bank data for public and publicly guaranteed (PPG) external debt for my set of EMs. Data is annual, and there is no data for Chile, Hungary, and Poland. I first compute the PPG external debt time series for each country, starting from 1997 to 2019. Then, I calculate the median for each country across time. Finally, I compute the median among the medians to find the value of 64.14%, which is my target.[8]

I discretize the exogenous VAR using the method developed by Tauchen (1986). I solve the model using Value Function Iteration. I guess a price and initial values. Given the price

[7] One alternative would be to run two separate AR(1) processes, one for output and another for inflation, and then estimate Σ_i for each i using the implied residuals. I include in the appendix the parameters estimated using this approach and also show other specifications for the price level and inflation.

[8] Table 9 in the Appendix show additional details.

schedule, I solve for value and policy functions using a grid search. Then, I check if the implied decision's price schedule is consistent with the price schedule used to compute the value and policies. I check for consistency in all policies, values, and price schedules. Further details on the grids and a more thoroughly explained of the steps to construct a solution are available in the Appendix.

4.2 Numerical Illustration

I solve the baseline and scenarios for the covariance matrix of innovation to exogenous process aiming at building up economic intuition. Below I describe the cases for which I solve the model and the simulation procedure that I use to compute moments for the model. In Appendix, I plot some objects of interest for the baseline case. Below I include plots comparing the baseline to parameterizations in which the covariance of interest is positive or negative.

In total, I solve the model for nine sets of parameters. The main exercise is to compare a baseline (1) that exhibits zero covariance between output and US dollar inflation to cases in which this covariance is positive (2) or negative (3). Then, I fix the zero covariance again and double, one at a time, the standard deviation of inflation (4) and output (5). Next, I return to the case in which the covariance is positive, compute the implied correlation between output and US dollar inflation, and double, one at a time, the volatility of inflation (6) and output (7). Finally, I repeat the same exercise but with a negative correlation, yielding another case with more volatile inflation (8) and output (9).

For each set of parameters, I solve for the recursive equilibrium. Then, equipped with policy functions, I simulate the model to compute moments. Here, I simulate each model 1,000 times for 1,000 periods. For each of the 1,000 simulations, I simulate the exogenous dynamics for $s \equiv (y, \pi)$ and feed the policies with these paths. I track every variable and the credit standing of the government. I follow the literature (e.g., Arellano (2008) and Chatterjee and Eyigungor (2012)) and compute moments based on periods in which the government is in good credit standing (not in default). I simulate 1,000 paths, calculate the moments for each course, and then average across paths.

For each simulation, I draw 1,200 observations according to the Markov Chain given the set of parameters and drop the first 200 observations to avoid the influence of initial conditions. The initial debt level is assumed at the target debt-to-output, 64.14%. Table 4 shows the computed moments for the baseline (1) and cases (2-5), while table 5 for the baseline (1) and cases (6-9). The baseline is included in both to serve as a benchmark.

Moment	$\sigma_{y\pi} = 0$	$\sigma_{y\pi} > 0$	$\sigma_{y\pi} < 0$	$2 \times \sigma_\pi$	$2 \times \sigma_y$
b/y	68.13	67.65	68.97	66.45	46.52
b'/y	68.19	67.72	69.02	66.50	46.64
$q \times b'/y$	67.45	66.98	68.28	65.79	46.04
$cor(q \times b', y)$	75.59	73.11	77.94	79.63	79.36
c/y	99.32	99.33	99.31	99.34	99.52
$std(c)$	2.99	3.06	2.90	2.94	5.01
$cor(c, y)$	74.89	74.14	76.18	76.57	83.94
nx/y	0.68	0.67	0.69	0.66	0.48
$cor(nx, y)$	-23.79	-24.64	-23.51	-25.21	-25.01
$spread$	0.40	0.46	0.35	0.34	1.24
$std(spread)$	0.43	0.48	0.38	0.35	1.14
$cor(spread, y)$	-23.80	-25.50	-12.43	-17.11	-0.74
$cor(spread, \pi)$	-5.57	-11.40	17.06	-0.46	-2.28
$\mathbb{E}_{s'\|s}[d(b', s')]$	0.10	0.11	0.08	0.08	0.29

Table 4: This table shows the simulated average of several moments. The first column shows the moments, the second column shows the baseline case in which the covariance is zero. The third column is the positive covariance, the fourth is the negative covariance. The last two column return to a zero covariance by double, one at a time, the standard deviation of innovations to inflation and the output, respectively. Here the net exports are given by $nx \equiv y - c$. std stands for standard deviation and cor stands for correlation.

In general, the economic intuition developed is accurate for this numerical illustration. If the covariance is positive, the government borrows less relative to the baseline and at worse spreads: they are higher and more volatile. The consumption volatility is higher than the baseline, and the government bears a riskier portfolio. In contrast, if the covariance is negative, the government can borrow more at higher prices — meaning better spreads, lower on average, and less volatile. This allows the government to disconnect consumption from income, rendering consumption less volatile.

For cases 4 to 9, the volatility of output or inflation is increased, one at a time, for various degrees of covariance between output and US dollar inflation. In general, increasing the volatility of either series worsens the borrowing conditions and impairs the government's ability to smooth out consumption when in good credit standing. The insurance emanating from US inflation when the covariance of interest is negative is increased whenever the variability of each series is heightened.

		$\sigma_{y,\pi} > 0$		$\sigma_{y,\pi} < 0$	
Moment	$\sigma_{y\pi}$	$2 \times \sigma_\pi$	$2 \times \sigma_y$	$2 \times \sigma_\pi$	$2 \times \sigma_y$
b/y	68.13	66.60	46.03	68.07	47.03
b'/y	68.19	66.67	46.15	68.12	47.15
$q \times b'/y$	67.45	65.93	45.56	67.39	46.54
$cor(q \times b', y)$	75.59	75.24	80.14	79.62	79.27
c/y	99.32	99.33	99.52	99.32	99.52
$std(c)$	2.99	3.11	5.02	2.79	4.98
$cor(c, y)$	74.89	74.35	84.18	77.50	83.90
nx/y	0.68	0.67	0.48	0.68	0.48
$cor(nx, y)$	-23.79	-25.96	-26.08	-22.83	-24.33
$spread$	0.40	0.46	1.21	0.35	1.21
$std(spread)$	0.43	0.50	1.12	0.25	1.13
$cor(spread, y)$	-23.80	-19.95	-0.02	-7.03	0.10
$cor(spread, \pi)$	-5.57	-13.34	-9.52	16.27	8.47
$\mathbb{E}_{s'\|s}[d(b', s')]$	0.10	0.11	0.29	0.08	0.29

Table 5: This table shows the simulated average of several moments. The first column shows the moments, the second column shows the baseline case in which the covariance is zero, for comparison. The third and fourth columns exhibit the cases in which the correlation is fixed at positive, and the volatility of innovations to inflation and output are doubled one at a time. The fifth and sixth columns repeat the exercise, but with the implied negative correlation. Here the net exports are given by $nx \equiv y - c$. std stands for standard deviation and cor stands for correlation.

Next, I include a few plots to compare value and policy functions under the three most interesting cases. In what follows, the black solid line shows the objects under the baseline parameters. The dotted red line exhibits the objects under the parameters associated with the positive covariance, and the dashed green line displays the objects under the negative covariance case. All plots take a particular combination, which is mean output $y = y_m = 1$

and inflation at its trend $\pi = \pi_m = 0$.

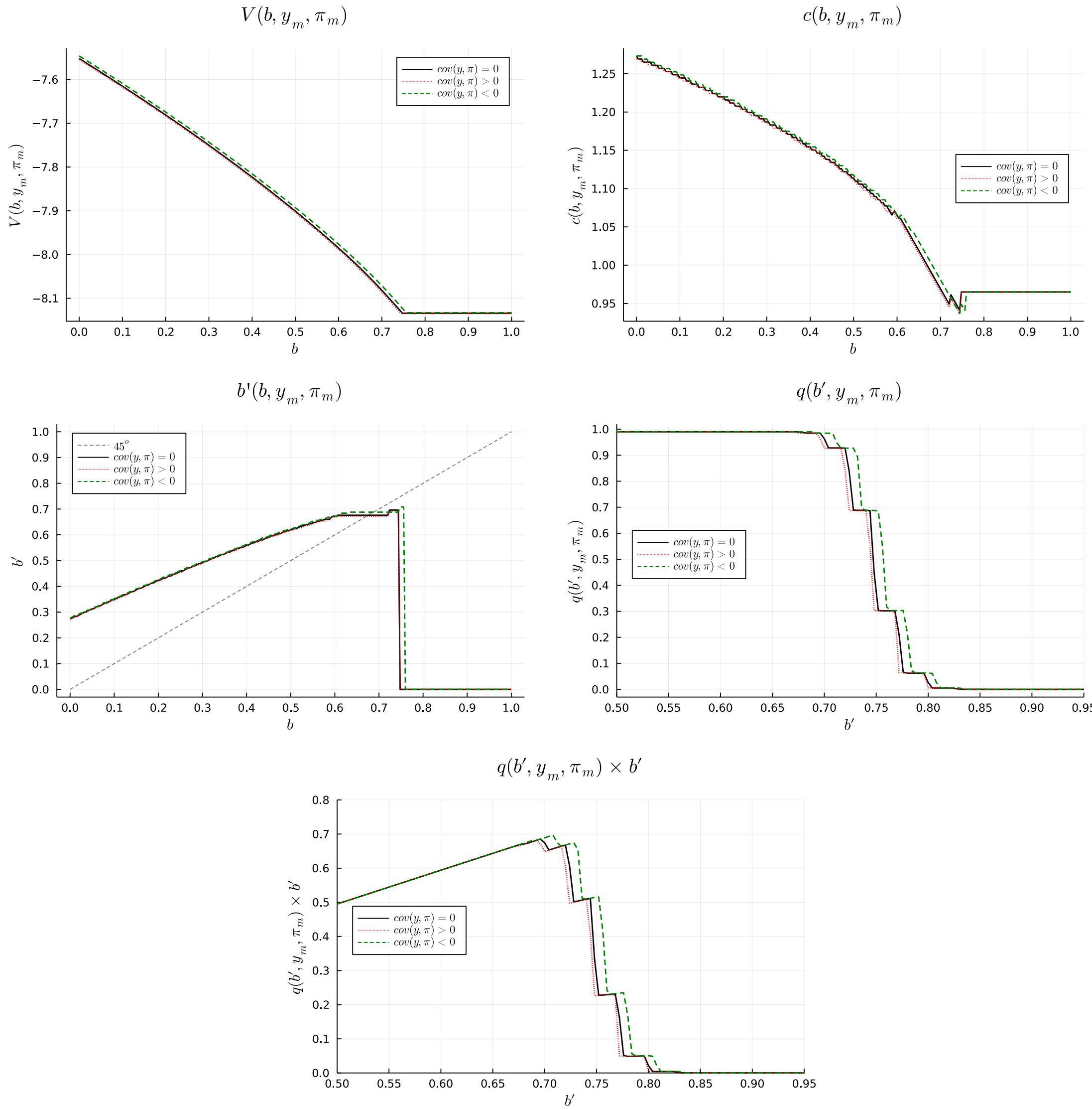

Figure 5: The figure display several objects of interest for three cases. In each plot, the solid black line in the figure is the baseline, the dotted red line is the positive covariance (2) and the dashed green line is the negative covariance (3). The first panel shows the Value function. The second panel shows the consumption policy. The third displays the borrowing policy. The last two bring the price schedule and the revenue from borrowing, respectively. In the pictures $y = y_m = 1$ and $\pi = \pi_m = 0$.

A general conclusion from the pictures is that having the output negatively correlated with US dollar inflation embeds the dollar-denominated borrowing with insurance properties. The value function is higher, and so is the consumption profile. The default region shrinks rendering favorable borrowing conditions. All these allow for lower spreads even with more indebtedness. These spreads are also less volatile and consumption is less volatile (under repayment).

5 Model Extension

In this section, I extend the simplified model that I developed first. I add a few features to the model to get it sufficiently rich to address the research question. In particular, I add: (i) the possibility for the government to issue local currency debt, adding one more endogenous state variable to the problem; (ii) the ability for the government to choose the nominal exchange rate, adding another choice variable for the government; (iii) disutility coming from inflation in domestic currency; (iv) long maturity bond.

Next, I describe each feature thoroughly and discuss why they are essential and the complications they bring to solving the model. Then, I put them together into a self-contained model.

5.1 Local Currency Debt

Adding local currency debt is probably the most essential feature of the baseline model. Since the research question pursued in this paper is whether or not the cross-country heterogeneity in the covariance between natural, local output, and US dollar inflation is informative about the cross-country heterogeneity on sovereign external debt composition, this paper cannot satisfactorily address this question without this feature.

I add local currency —LC— debt to the baseline model to capture the ability of the sovereign to borrow in their domestic currency. This power comes with additional enforcement problems. An important issue is that an ex-post devaluation of the domestic currency alleviates the debt repayment burden, making it desirable for the government to engage with such arbitrarily large depreciation ex-post. Hence, one needs to look for Time Consistent nominal exchange rate policies.

Adding LC debt enlarges the state space, summing up to four — and two of them are endogenous states. There are two main complications arising from this. First, it simply makes the curse of dimensionality issue self-evident. Any reasonable solution would quickly approach one million points in the state space.

Second, and more related to the paper itself, with two debt alternatives, the government faces a dynamic portfolio decision to address. If these bonds are long-term, the government might want to dilute the non-maturing stock of debt by issuing more debt in the current period. Hence, with a portfolio of long maturity, there are cross-side effects of currency-specific issuance, rendering the problem much harder to solve than the plain vanilla long-term single bond problem.

5.2 Exchange Rate Regime

Several recent papers have emphasized that the choice of exchange rate regime might put the country under difficult circumstances when the government borrows from abroad (e.g., Schmitt-Grohé and Uribe (2016), Na et al. (2018), Bianchi and Sosa-Padilla (2020), Bianchi and Mondragon (2022), and Du and Schreger (2022)). These papers emphasize that a fixed exchange regime can turn nominal friction into real friction, exacerbating income fluctuations and rendering consumption more volatile than if the country adopted a freely floating exchange rate.

In practice, the government does have some control over the nominal exchange rate, and there are exciting problems arising from models in which the government can directly pick the value for this object. When the government can choose the exchange rate ex-post, Time Consistency becomes a critical issue because the devaluations erode the real value of debt due to foreigners, and the government can use this instrument to default on its obligations partly.

While erosion through inflation in the US dollar is present in a simplified model, that happens exogenously. On the other hand, the government chooses the ex-post dilution endogenously through nominal exchange rate depreciations. Therefore, if the covariance of interest is unfavorable (positive), then it could be the case that the government would try to use the exchange rate to distort this covariance using LC debt aiming at smoothing out income fluctuations.

5.3 Inflation Cost

If there is a single good and the Law of One Price holds, given the dynamics for the inflation in US dollars, the government can directly choose the domestic inflation rate by selecting the exchange rate dynamics. Hence, as emphasized by Ottonello and Perez (2019), without any costs to domestic inflation, the government has the incentives to set an arbitrarily large ex-post depreciation to eliminate LC debt.

The government can never commit to some agreed-upon depreciation rate because it cannot commit to repaying debts to foreigners. Even though there might be costs associated with specific choices, the government always has the choice to pick the option that yields the highest value. Inflation costs here are essential to bound the willingness of the government depreciation ex-post. Without any domestic currency inflation costs, the government could not commit to not expropriating all the value of debt through depreciation, and the international lenders, forecasting this behavior, would never lend any positive amount denominated in LC.

A direct implication of this argument is that the simplified model is, in fact, a particular case of the extended model in which the inflation costs are zero at all times and in all states.

5.4 Long-term Bond

As formalized by Cao and Gordon (2019), a sovereign default model of short-term bonds cannot generate the joint dynamics of sovereign indebtedness and spreads. This claim has been made and explored elsewhere in the literature, e.g., Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012).

This feature brings the model close to the data. While in late 1980 and 1990, the maturity of external sovereign debt was short, closer to two years, it has never been close to a single quarter. The quarterly period and short-term debt model implies that the government has to, conditional on repayment, roll over a significant fraction of income in the form of debt, which is implausible to the levels observed in the data.

Making the debt longer term gets the model closer to the observed average maturity in the data. At the same time, the longer debt allows the government to dilute the non-maturing fraction of debt by increasing the debt choice in the future. This feature leaves room for Time Inconsistency, as emphasized elsewhere in the literature, e.g., Hatchondo et al. (2016), making the model harder to solve but rendering non-obvious dynamics on borrowing over the business cycles.

5.5 Extended Model

In what follows, variables with * superscript denote variables denominated in US dollars. I represent the nominal exchange rate by e_t as the LC price of one FC unit.

There are four state variables: (b^*, b, y, π^*). The first is the debt denominated in US dollars, and the second is the amount of debt denominated in local currency, both endogenous. The third is the endowment of real output; the fourth is the inflation in US dollars. These are exogenous. As before, the price level normalizes the nominal debt level, here denominated in LC units.

These debts are long-term. Following Hatchondo and Martinez (2009), I assume that bonds pay a deterministic infinite stream of coupons that decreases over time. Effectively, a bond issued at date t pays, in period $t + j$, for $j \geqslant 1$

$$\delta(1 - \delta)^{j-1} \tag{24}$$

It follows that the bond's duration is given by δ^{-1}, which we assume for simplicity is the same for both LC and FC debt. For new issuance I_t^* and I_t, for FC and LC debt, respectively, the face value of debt dynamics obeys the following law of motion in case of repayment

$$B_{t+1}^* = (1-\delta)B_t^* + I_t^* \tag{25}$$

$$B_{t+1} = (1-\delta)B_t + I_t \tag{26}$$

In this framework, one does not need to keep track of every bond ever issued but instead only of the face value of debt for each type. Effectively, one needs to track only two state variables (after normalization): (b_t^*, b_t). For simplicity, I assume that the maturing fraction δ of each debt is the same for both types of debt.

The prices of these bonds at date t are q_t^* and q_t. Equilibrium prices depend on the repaying incentives for the government, the contingent choices of the government to devaluate the domestic currency, and outside options for lenders.

Whenever the government defaults, the household faces a disutility cost, which I assume is a strictly increasing function of the current endowment level of output. Precisely, I assume that when the government is under default, the household utility deducts the following term:

$$\psi(y_t) \geqslant 0, \qquad \psi_d' \geqslant 0 \tag{27}$$

So everything else is constant, it is more costly to default in good times. This assumption leads to countercyclical incentives to default and resulting countercyclical bond spreads—consistent with patterns observed in the data.

This assumption is in line with the idea developed by Bianchi et al. (2018). In that paper, the disutility cost is used instead of the output cost in case of a default. There are mainly two reasons. First, one cannot observe output costs, so imposing disutility costs is not entirely arbitrary. Depending on the utility function and the penalty function for output, utility and output costs are interchangeable. Second, and more importantly, the disutility cost does not affect the budget of the government constraint under default. For these reasons, utility cost has become the dominant approach in the literature.

A secondary cost of default is a temporary exclusion from financial markets. I assume that conditional on being on default today, the government will regain access to financial markets tomorrow with a probability θ, which is i.i.d. across time and independent of the exogenous state $s \equiv (y, \pi^*)$.

The government's budget constraint depends on whether it has defaulted or not at the current period and on whether the government has access to international markets. The reason is that the government could have defaulted in the past but not yet regained access to global financial markets in the current period.

In recursive form, the default decision depends on the four states, as follows

$$d(b^*, b, y, \pi^*) = \begin{cases} 0 & V^d(b^*, b, y, \pi^*) \leqslant V^r(b^*, b, y, \pi^*)\text{t} \\ 1 & V^d(b^*, b, y, \pi^*) > V^r(b^*, b, y, \pi^*)\text{t} \end{cases} \tag{28}$$

where $V^r(b^*, b, y, \pi^*)$ is the repayment value under state (b^*, b, y, π^*), and $V^d(b^*, b, y, \pi^*)$ is the default value under state $(0, 0, y, \pi^*)$.

In case the government is in good credit standing, its budget constraint in LC is given by

$$P_t c_t + \delta e_t B_t^* + \delta B_t \leqslant P_t y_t + e_t q_t^* I_t^* + q_t I_t \tag{29}$$

Using the law of motion for the face value of each type of debt, conditional on repayment, one can rewrite the budget constraint as follows

$$P_t c_t + \delta e_t B_t^* + \delta B_t \leqslant P_t y_t + e_t q_t^* (B_{t+1}^* - (1-\delta) B_t^*) + q_t (B_{t+1} - (1-\delta) B_t) \tag{30}$$

The right-hand side of this equation shows the government's uses sources, with two components in addition to the endowment: domestic currency value of net issuance for both the FC debt and the LC debt. The left-hand side of this equation shows the sources of resources by the government. First, there is the endowment. Then, the value for the net issuance of debt denominated in both FC and LC.

When the government is in bad credit standing, its budget constraint becomes much simpler for two reasons. First, existing debt vanishes, that is $B_t^* = B_t = 0$. Second, because the government is excluded from the financial markets, net issuance is constrained to be zero: $I_t^* = I_t = 0$. Hence, the budget constraint is simply

$$P_t c_t \leqslant P_t y_t \tag{31}$$

So that under bad credit stands, the government consumes the realized endowment for output.

I assume the endowment good is freely traded in international markets, so the Law of One Price (LOOP) holds, implying that the real exchange rate is equal to one at every period and every state:

$$P_t = e_t P_t^* \tag{32}$$

where P_t^* is the price level denominated in FC currency. By construction, the dynamics of π_t^* drives the dynamics of P^*. The dynamic version of the LOOP gives rise to the link between the devaluation of the local currency and domestic inflation, as follows

$$1 + \pi_t = (1 + \varepsilon_t)(1 + \pi_t^*) \tag{33}$$

where $1 + \varepsilon_t \equiv \frac{e_t}{e_{t-1}}$ gives the gross depreciation of the domestic (or local) currency. It is clear that, given a level of realized US dollar inflation, the government chooses π_t directly by choosing ε_t because of equation 33.

Since the government can evaluate ex-post and wipe away part of the real value of its obligations in local currency, there must be some inflation cost — or free of floating, as highlighted elsewhere, e.g., in Calvo and Reinhart (2002). Hence, I assume that the government faces convex costs of inflation or deflation, away from zero.[9]

$$\phi(\pi_t) \geqslant 0, \qquad \phi(0) = 0, \qquad \frac{\partial^2 \phi}{\partial \pi^2} \geqslant 0 \tag{34}$$

International lenders are risk-neutral and have deep pockets. They discount real flows by $1 + r$, and their break-even price considers expected inflation, default, and the devaluation of the local currency in the case of debt denominated in local currency. A critical piece of dilution of the current price is due to future borrowing decisions, in the future. Choosing high borrowing for the next period tends to increase the likelihood of future default, reducing the bond price. But this is the same price for all the non-maturing fractions of debt.

The bond price of the debt denominated in FC is given by

$$q^*(b^{*\prime}, b', y, \pi^*) = \frac{1}{R}\mathbb{E}_{s'|s}\left[\frac{1 - d(b^{*\prime}, b', y', \pi^{*\prime})}{1 + \pi^{*\prime}}\left(\delta + (1-\delta)q^*(b^{*\prime\prime}, b'', y', \pi^{*\prime})\right)\right] \tag{35}$$

and the bond price of the debt denominated in LC is given by

$$q(b^{*\prime}, b', y, \pi^*) = \frac{1}{R}\mathbb{E}_{s'|s}\left[\left(\frac{1 - d(b^{*\prime}, b', y', \pi^{*\prime})}{1 + \pi^{*\prime}}\right)\left(\frac{1}{1 + \varepsilon(b^{*\prime}, b', y', \pi^{*\prime})}\right)\left(\delta + (1-\delta)q(b^{*\prime\prime}, b'', y', \pi^{*\prime})\right)\right] \tag{36}$$

where the price take into account the depreciation of the nominal exchange rate at the future state, denoted by $\varepsilon(b^{*\prime}, b', y', \pi^{*\prime})$.

The problem of the government first deciding whether or not to default,

$$V(b^*, b, y, \pi^*) = \max_{d \in \{0,1\}} \left\{(1-d)V^r(b^*, b, y, \pi^*) + dV^d(y, \pi^*)\right\} \tag{37}$$

[9] I assume that zero inflation is cost-free because I work on cyclical inflation, not gross inflation itself. If I were to work with inflation itself, the inflation associated with no inflation cost would be the average inflation observed in the data or a targeted inflation $\bar{\pi}$. If the specification were of the fear of floating costs, the argument of the function would be ε_t instead of π_t.

and where V^r denotes the value under repayment and V^d denotes the value under default.

The value under default is given by

$$
\begin{aligned}
V^d(y,\pi^*) = \max_{c,\varepsilon} & \left\{ u(c) - \psi(y) - \phi(\pi) + \beta \mathbb{E}_{s'|s} \left[(1-\theta) V^d(y',\pi^{*\prime}) + \theta V(0,0,y',\pi^{*\prime}) \right] \right\} \\
& \text{subject to} \qquad\qquad (38) \\
& c \leqslant y \\
& (1+\pi) = (1+\pi^*)(1+\varepsilon)
\end{aligned}
$$

The government can choose how much to consume and how much to devalue the local currency. There are two utility costs: one for being in default and another one for having inflation, associated with the devaluation through the dynamic law of one price.

The value under repayment is given by

$$
\begin{aligned}
V^r(b^*,b,y,\pi^*) = \max_{c,\varepsilon,b^{*\prime},b'} & \left\{ u(c) - \phi(\pi) + \beta \mathbb{E}_{s'|s} \left[V(b^{*\prime},b',y',\pi^{*\prime}) \right] \right\} \\
& \text{subject to} \qquad\qquad (39) \\
c + \delta \frac{b^*}{1+\pi^*} + \delta \frac{b}{1+\pi} \leqslant y + q^* \times & \left(b^{*\prime} - (1-\delta) \frac{b^*}{1+\pi^*} \right) + q \times \left(b' - (1-\delta) \frac{b}{1+\pi} \right) \\
& (1+\pi) = (1+\pi^*)(1+\varepsilon) \\
& q^* \equiv q^*(b^{*\prime},b',y,\pi^*) \\
& q \equiv q(b^{*\prime},b',y,\pi^*)
\end{aligned}
$$

The last two identities clarify that the price schedules q^*, q depend on future borrowing decisions and the current exogenous state. I shortened the notation in the first constraint (budget constraint) to save space.

The government has four choices. How much to consume c, to devalue the domestic currency ε, and to borrow for the future using debt in FC $b^{*\prime}$ and denominated in LC b'. Conditional on repayment, it is essential to realize that dilution through inflation affects both types of debt differently. While debt denominated in FC (US dollars) is diluted ex-post through inflation in the FC currency, which happens exogenously, the dilution of debt denominated in LC occurs endogenously, as the government can choose ε directly. These incentives will be priced in by international lenders.

Lastly, I define the recursive equilibrium for the extended model. Because of the existence of dilution throughout future debt issuance, and thus continuation price schedule, in addition to dilution employing a devaluation of the LC currency, I specialize in Markov Perfect Equilibrium. In such equilibrium, government and international lenders take as given the actions played by future governments, and current actions depend only on current state variables but not on expectations about the actions still to be taken by future governments. The payoff relevant state is (b^*, b, s) where $s \equiv (y, \pi^*)$. Also, there is a constraint on the perceived (by the international lenders) law of motion of the nominal exchange rate depreciation. I follow Ottonello and Perez (2019) and call this a Time Consistency constraint.

Definition 5.1 (Markov Perfect Equilibrium) The Markov Perfect Equilibrium is a set of

a) Value functions $\left\{V(b^*, b, s), V^d(s), V^r(b^*, b, s)\right\}$;

b) Associated policy functions for $\left\{d(b^*, b, s), c(b^*, b, s), b^{*\prime}(b^*, b, s), b'(b^*, b, s), \varepsilon(b^*, b, s)\right\}$;

c) Two Price Schedules $q^*(b^{*\prime}, b', s)$ and $q(b^{*\prime}, b', s)$; and

d) A perceived (by the international lenders) law of motion for nominal exchange rate depreciation $\hat{\varepsilon}^{IL}(b^*, b, s)$

such that

I) Taking as given the price schedules, the policies solve the government's problem

II) The price schedules are consistent with break-even for the international lenders

III) (Time Consistency) The perceived law of motion for the nominal exchange rate is correct:

$$\hat{\varepsilon}^{IL}(b^*, b, s) = \varepsilon(b^*, b, s), \qquad \forall(b^*, b, s) \tag{40}$$

Where equation 40 highlights the time consistency issues arising from the incentives to ex-post depreciate the local currency to dilute the existing debt.

5.6 Discussion and Special Cases

As discussed before, the case of the extended model in which the cost of inflation in local currency is zero, that is $\phi(\pi) = 0, \forall \pi$, implies a complete shutdown of the LC debt. Ottonello and Perez (2019) established this result in a simplified environment.

A general implication is that whenever the government does not face any cost of inflation in its domestic currency, it can only borrow from international lenders in some other currency. Hence, the relevant budget constraint for the government is denominated in the currency from which the government borrows. The simplified model should be seen in this case, and the government borrows US dollars.

There are other interesting special cases too. One, similarly explored by Ottonello and Perez (2019), takes the dynamics of the nominal exchange rate as exogenous. In this case, if the correlation between output and the nominal exchange rate is negative, there is a hedging value for LC debt. The central intuition is that if the nominal exchange rate is countercyclical, the devaluation of the LC lowers the burden of the debt denominated in this currency but measured in real terms in states where the government is facing a low realization of about, and therefore default incentives are high. The two forces pushing the cash-in-hand of the government in different directions provide a stabilizing force and tend to deliver better borrowing conditions.

Another interesting limiting case is the one in which the government faces an infinity cost of inflation in LC, that is $\phi(\pi) = \infty, \forall \pi$, but the exchange rate is endogenous. Since

US dollar inflation contaminates the LC inflation by the dynamic LOOP, the government must use the exchange rate to counter this. Inflation in LC becomes zero everywhere to eliminate any risk from the ex-post dilution and, maybe, improve borrowing conditions for debt denominated in LC. It remains to explore what would happen to the default decision under this scenario.

Finally, another special case considers the fear of floating as an alternative to the cost of inflation denominated in LC. The main trade-off would change substantially relative to the baseline extended model. In the baseline extended model, the government needs to use the exchange rate to counterbalance the inflation denominated in FC to avoid generating inflation in LC. Since the sovereign does not need to use the nominal exchange rate to directly counterbalance US dollar inflation, aiming to avoid LC inflation, with the fear of floating cost, inflation in FC would become less relevant to shape the devaluation decision. But, this approach prevents arbitrarily large devaluation ex-post.

5.7 Limitations and Avenues for Future Research

I see four prominent limitations of this extended model. First, recent evidence has explored maturity extensions to manage debt portfolios in EMs, e.g., Arellano et al. (2013), Arellano and Ramanarayanan (2012), Dvorkin et al. (2021). The setup of the extended model does not allow for that. For instance, it could be optimal for the government to choose a shorter debt maturity when output faces a recession. There is sparse evidence that the opposite happened during the commodity boom before the Global Financial Crisis. During this period, debt maturity generally became longer during this high output realization for several EMs.

A second critical limitation is the assumption of defaultability of debt denominated in LC. Du and Schreger (2016) shows a sizable premium for EMs to borrow in LC, and they attribute this partly to default risk. In practice, few outright defaults have been observed for sovereign debt denominated in LC. It seems more plausible to allow default only of debt denominated in FC and model premium for debt denominated in LC as liquidity costs or information acquisition costs.

A third noteworthy limitation of the analysis is considering only a single good model. A model with two goods —tradable and non-tradable — would generate dynamics of the real exchange rate (RER) with further spillovers emanating from the nominal frictions. In the extended model, the RER is fixed at one in every period and every state. It became standard in this literature to work with models that feature a tradable and a non-tradable sector, e.g., Bianchi (2011), Schmitt-Grohé and Uribe (2016), Na et al. (2018), Bianchi and Sosa-Padilla (2020), Ottonello and Perez (2019). The latter works out with a simplified model to show that the government might want to boost the consumption of tradable or non-tradable goods to distort the RER and improve the borrowing terms. The pricing of bonds denominated in both currencies would become more complicated, and the economic intuition developed before might be insufficient, rendering the analysis primarily quantitative.

Finally, the model works in an endowment economy. As Arslanalp and Tsuda (2014) emphasizes, one of the critical reasons to understand the rise of Ems's sovereign external indebtedness denominated in LC is the potential implications to the borrowing costs for firms and banks. Bocola (2016) and Du and Schreger (2022) provide informative work in this direction. A model with production would become much harder to solve and understand the main channels of action. A related limitation is that I take the covariance between real, local output, and the US dollar as exogenous. Lastly, the current environment also lacks a key feature of the data, that the real interest rate fluctuates over time, in the spirit of the Taylor Principle (e.g, Taylor (1993)). Adding these features would provide a meaningful way to endogenize such covariance at the cost of rendering the model less transparent and substantially harder to solve.

6 Conclusion

This paper showed sizable dispersion in the measured covariance between real, local output, and US dollar inflation. Next, I show that the higher this covariance is, the higher on average and more volatile spreads for external debt denominated in US dollars tend to be. The evidence also suggests that countries with higher covariance tend to have a larger share of their debt denominated in LC owed to foreigners, which increased the most between 2009 and 2019.

With this evidence in hand, I forged a simple sovereign default model in which the price level of the currency that denominated the government's debt is random. The government faces different borrowing opportunities depending on how real output and inflation in this foreign currency relate over time. In particular, if the covariance between these two objects is positive, there will be significantly more variability of resources available to the country in the next period, fixing the amount of debt, and hence worse borrowing opportunities, in line with the data patterns documented earlier.

I next enrich the simple model by allowing the government to choose the currency composition of its outstanding debt owed to foreigners. In such a setup, the government faces further incentive problems. An ex-post dilution employing a nominal devaluation of the domestic currency— interpreted as partial default— makes the government face more profound Time Inconsistency problems. I discuss the assumptions and analyze exceptional cases, arguing that the simple model is a particular parametrization of the extended model.

Finally, I reconnect other limiting cases of the extended model to alternative interpretations and the existing literature. I emphasize the limitations of the larger model and highlight the potential avenues for future research.

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A Numerical Algorithm

Below I describe the steps to solve the model numerically.

1. Define parameters and construct a grid for b, $B \equiv [0,1]$, $Nb = 251$

2. Discretize the VAR(1) using the method developed in Tauchen (1986)

   $3\times$ the unconditional standard deviation of y and π

   tensor grid for $s \equiv (y,\pi)$, $Ns = 11 \times 11 = 121$

   $\Pi(s'|s)$ transition matrix

3. Set a relaxation parameter $\xi_q \in (0,1]$ and $t = 1$

4. Guess initial values and price, for all $(b,s) \in (B,S)$

   Values: $V_{t-1}(b,s) = V_{d,t-1}(s) = V_{r,t-1}(b,s) = 0$;

   Price: $q_{t-1}(b,s) = \frac{1}{1+r}\mathbb{E}_{s'|s}\left[\frac{1}{1+\pi(s')}\right]$

4. Given $q_{t-1}, V_{t-1}, V^d_{t-1}, V^r_{t-1}$, solve for:

   values: $V_t(b,s)$, $V^d_t(s)$, $V^r_t(b,s)$

   policies: $c_t(b,s)$, $b'_t(b,s)$, $d_t(b,s)$

   associated price $\tilde{q}_t(b,s)$

5. Update price of bond:

$$q_t(b,s) = (1-\xi_q)q_{t-1}(b,s) + \xi_q\tilde{q}_t(b,s)$$

6. For each $\psi \in \Psi \equiv \{q, V, V^d, V^r, c, b', d\}$ get

$$\varepsilon(\psi) \equiv \max_{(b,s)\in(B\times S)} \|\psi_t(b,s) - \psi_{t-1}(b,s)\|$$

7. If following condition is not satisfied, let $t-1=t$ go back to step 4

$$\max_{\psi \in \Psi} \quad \varepsilon(\psi) < 10^{-8}$$

B Discretizing the VAR(1)

In this appendix, I show how the discretization procedure by Tauchen (1986) works and give two illustrative examples. I experimented with the improvements proposed by Gordon (2021), but the procedure proposed did not eliminate sufficient grid points to justify the lack of clarity that would come with its adoption.

Consider the following system

$$z_t = \mathbf{c} + A z_{t-1} + \eta_t$$

where z and c are $D \times 1$ and A is $D \times D$, $\eta \sim \mathbf{N}(0, \Sigma)$. For the case of uncorrelated shocks, Σ is diagonal, while for correlated ones, Σ is not diagonal.

The first task is to decompose the real, symmetric matrix Σ using the spectral decomposition as follows:

$$\Sigma = L \Lambda L^T$$

where L is orthogonal, i.e. $L^T L = I$ and Λ is diagonal.

Next, use the L matrix to rotate the system and define a new system with transformed variables:

$$\tilde{z} \equiv L^T z, \qquad \tilde{c} \equiv L^T c, \qquad \tilde{A} \equiv L^T A L, \qquad \tilde{\eta} \equiv L^T \eta$$

yielding the transformed system by pre-multiplying the original one by L^T

$$\tilde{z}_t = \tilde{c} + \tilde{A} \tilde{z}_{t-1} + \tilde{\eta}_t$$

where $Var(\tilde{\eta}) = L^T \Sigma L = L^T L \Lambda L^T L = \Lambda$, following that $\tilde{\eta} \sim N(0, \Lambda)$

Now, compute the unconditional Var-Covar and Expected value $\tilde{z}$, denoted by $\tilde{V}$ and $\tilde{E}$:

$$vec(\tilde{V}) = \left(I - \tilde{A} \otimes \tilde{A}\right)^{-1} vec(\Lambda)$$

$$\tilde{E} = \left(I - \tilde{A}\right)^{-1} \tilde{c}$$

For each $d \in D$, set a coverage κ_d and get the univariate grid.

$$\tilde{Z}_d \equiv \tilde{E}_d \pm \kappa_d \tilde{V}_{d,d}^{1/2}$$

Get the tensor grid

$$\tilde{Z} \equiv \bigtimes_{d=1}^{D} \tilde{Z}_d$$

We are now in place to compute the transition matrix for $\forall \tilde{z}_j, \tilde{z}_i \in \tilde{Z}$:

$$\Pi(\tilde{z}_j | \tilde{z}_i)$$

Compute the transition probabilities using the conditional distribution

$$\tilde{z}_i \in \tilde{Z} \rightarrow \tilde{z}_j \in \tilde{Z} \qquad \mathbf{N}(\tilde{c}_d + \tilde{A}_{(d,\cdot)} \tilde{z}_i, \Lambda_d)$$

Back out the grids for the original system

$$Z \equiv \left\{ L\tilde{z} \,|\, \tilde{z} \in \tilde{Z} \right\}$$

$$\text{since} \qquad L\tilde{z} = LL^T z = Iz = z$$

If Σ is not diagonal, then $L \neq I$, and the grid will be rectangular for $\tilde{z}$ but a rotation (or a linear transformation of it) for z. The two examples below highlight that feature.

B.1 Example with $cov(y,\pi) > 0$

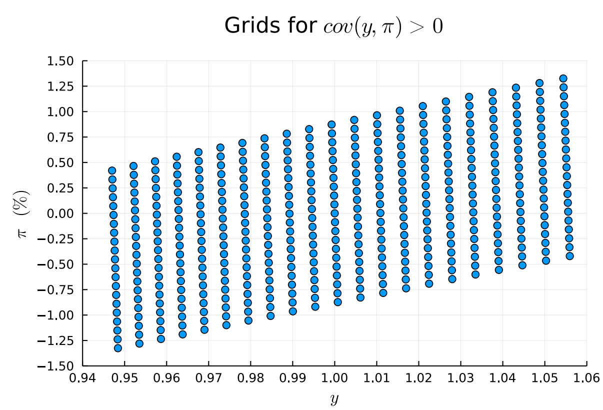
Consider that

$$\Sigma \equiv 10^{-4} \times \begin{bmatrix} 1.0063^2 & 0.0858 \\ 0.0858 & 0.1964^2 \end{bmatrix}$$

The spectral decomposition of Σ yields:

$$L = \begin{bmatrix} -0.996201 & 0.0870792 \\ -0.0870792 & -0.996201 \end{bmatrix} \qquad \Lambda = 10^{-4} \times \begin{bmatrix} 1.02014 & 0.00 \\ 0.00 & 0.03107 \end{bmatrix}$$

And the resulting grid for z (denoted by as s in the main text) will be given by



B.2 Example with $cov(y,\pi) < 0$

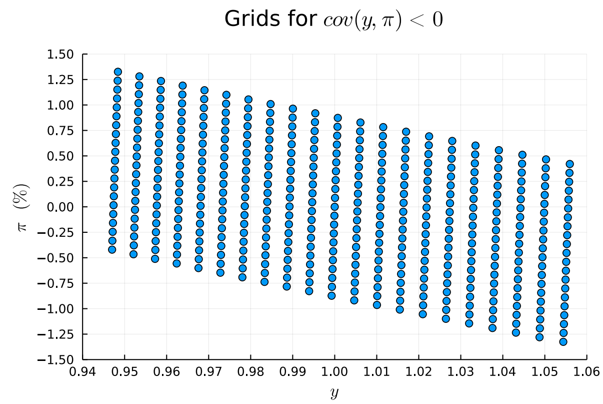
Consider that

$$\Sigma \equiv 10^{-4} \times \begin{bmatrix} 1.0063^2 & -0.0858 \\ -0.0858 & 0.1964^2 \end{bmatrix}$$

The spectral decomposition of Σ yields:

$$L = \begin{bmatrix} -0.996201 & -0.0870792 \\ 0.0870792 & -0.996201 \end{bmatrix} \qquad \Lambda = 10^{-4} \times \begin{bmatrix} 1.02014 & 0.00 \\ 0.00 & 0.03107 \end{bmatrix}$$

And the resulting grid for z (denoted by as s in the main text) will be given by


Grids for $cov(y, \pi) < 0$
1.50
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
π (%)
0.94 0.95 0.96 0.97 0.98 0.99 1.00 1.01 1.02 1.03 1.04 1.05 1.06
y

C Alternative Specifications for the VAR

In this appendix, I show additional estimates for a few different specifications for the VAR in section 2, "Empirical Support". In particular, I run three other specifications. In all of them, I use the same series (for each EM in the sample) for the cyclical component of output.

First, I consider the measured inflation as the quarterly change in the logged price level. Table 6 exhibit the estimates.

Next, I consider the cyclical component of the price instead of the cyclical component of inflation. Table 7 contains the estimates.

Finally, I run two separate AR(1) for output and the cyclical component of US inflation, as defined in equation 3. Then, I compute the variance-covariance matrix Σ using the residuals from both exercises. This is equivalent to estimating 4 imposing $\rho_{y\pi} = 0$ and $\rho_{\pi y} = 0$. The main goal of this exercise is to prevent any "cross memory" between series (e.g., prevent the output of Indonesia be a (potentially powerful) forecaster of US dollar inflation). Another good reason for that is that the estimated variances (and covariances) would then increase in absolute value, as otherwise, in the baseline specification, the unrestricted parameters $\rho_{y\pi}$ and $\rho_{\pi y}$ "filter" out partially the noise in ϵ_y and ϵ_π. Table 8 highlights the final estimates.

Overall, the alternative specifications suggest that Ems's cyclical output is not informative about the US inflation and several of its measures. On the contrary, some measurements of the US dollar inflation do have some forecasting power for the cyclical component of the output for some EMs. This could be due to the fact there might be "global" shocks affecting primarily US inflation and its various measures together with Ems's cyclical output.

Country	ρ_{yy}	$\rho_{\pi y}$	$\rho_{y\pi}$	$\rho_{\pi\pi}$	σ_y	$\sigma_{y\pi}$	σ_π
Argentina	0.86	0.14	0.00	0.89	1.88	0.07	0.25
Brazil	0.80	0.07	-0.01	0.89	1.02	0.03	0.25
Chile	0.83	0.04	-0.01	0.89	1.01	0.03	0.25
Colombia	0.82	0.07	-0.01	0.88	0.83	0.00	0.25
Costa Rica	0.78	0.05	-0.01	0.89	0.84	0.01	0.25
Mexico	0.82	0.15	-0.02	0.90	0.92	0.05	0.25
Bulgaria	0.59	0.12	-0.01	0.88	1.87	-0.03	0.25
Hungary	0.84	0.15	-0.01	0.89	0.77	0.00	0.25
Poland	0.69	0.01	-0.02	0.89	0.86	0.02	0.25
Romania	0.86	-0.02	-0.01	0.88	1.18	0.05	0.25
Russia	0.85	0.16	-0.01	0.89	1.36	0.04	0.25
Turkey	0.81	0.06	0.00	0.88	1.97	0.07	0.25
South Africa	0.88	0.08	-0.02	0.89	0.47	0.00	0.25
India	0.80	-0.06	0.02	0.88	0.88	0.01	0.25
Indonesia	0.76	-0.03	-0.01	0.88	1.34	0.01	0.25
mean	0.80	0.07	-0.01	0.89	1.15	0.02	0.25
minimum	0.59	-0.06	-0.02	0.88	0.47	-0.03	0.25
median	0.82	0.07	-0.01	0.89	1.01	0.02	0.25
maximum	0.88	0.16	0.02	0.90	1.97	0.07	0.25
standard deviation	0.07	0.07	0.01	0.00	0.44	0.03	0.00

Table 6: VAR estimated from equation 4 using the cyclical component of the output for each EM and (net) US dollar inflation rate instead of the cyclical component of the US dollar inflation. The net inflation rate is the first difference of the logged US dollar price level, measured by the US GDP deflator.

Country	ρ_{yy}	$\rho_{\pi y}$	$\rho_{y\pi}$	$\rho_{\pi\pi}$	σ_y	$\sigma_{y\pi}$	σ_π
Argentina	0.91	-1.00	0.02	0.83	1.83	0.06	0.18
Brazil	0.83	-0.22	0.05	0.82	1.02	0.04	0.18
Chile	0.85	-0.13	0.07	0.75	1.01	0.04	0.18
Colombia	0.75	0.37	0.02	0.87	0.82	0.03	0.20
Costa Rica	0.77	0.09	0.03	0.86	0.84	0.02	0.19
Mexico	0.90	-0.45	0.06	0.80	0.91	0.06	0.18
Bulgaria	0.51	1.06	-0.01	0.91	1.81	0.02	0.20
Hungary	0.86	-0.05	0.07	0.77	0.78	0.03	0.18
Poland	0.69	0.03	0.02	0.89	0.86	0.03	0.20
Romania	0.78	0.67	0.00	0.91	1.15	0.06	0.20
Russia	0.87	-0.14	0.03	0.78	1.36	0.08	0.19
Turkey	0.86	-0.89	0.03	0.81	1.93	0.04	0.17
South Africa	0.87	0.01	0.13	0.67	0.47	0.02	0.18
India	0.80	-0.37	0.06	0.89	0.86	-0.01	0.18
Indonesia	0.76	0.01	0.00	0.90	1.34	0.04	0.20
mean	0.80	-0.07	0.04	0.83	1.13	0.04	0.19
minimum	0.51	-1.00	-0.01	0.67	0.47	-0.01	0.17
median	0.83	-0.05	0.03	0.83	1.01	0.04	0.18
maximum	0.91	1.06	0.13	0.91	1.93	0.08	0.20
standard deviation	0.10	0.51	0.03	0.07	0.42	0.02	0.01

Table 7: VAR estimated from equation 4 using the cyclical component of the output for each EM and the cyclical component of the US dollar price level instead of the cyclical component of the US dollar inflation.

Country	ρ	σ	$cov(y, \pi^c)$	$cor(y, \pi^c)$
Argentina	0.87	1.86	0.06	0.16
Brazil	0.79	1.00	0.03	0.14
Chile	0.79	0.97	0.02	0.11
Colombia	0.83	0.83	-0.01	-0.05
Costa Rica	0.79	0.83	0.01	0.08
Mexico	0.83	0.91	0.03	0.14
Bulgaria	0.59	1.85	-0.03	-0.09
Hungary	0.85	0.77	0.00	-0.03
Poland	0.69	0.85	0.02	0.12
Romania	0.87	1.18	0.00	0.02
Russia	0.87	1.36	0.00	-0.01
Turkey	0.81	1.95	0.08	0.21
South Africa	0.90	0.47	-0.02	-0.16
India	0.76	0.85	0.03	0.18
Indonesia	0.85	1.40	-0.01	-0.03
π^c	0.21	0.20	0.04	1.00
mean	0.77	1.08	0.02	0.11
minimum	0.21	0.20	-0.03	-0.16
median	0.82	0.94	0.02	0.10
maximum	0.90	1.95	0.08	1.00

Table 8: Restricted VAR estimated from equation 4 using the cyclical component of the output for each EM and the cyclical component of the US dollar inflation, here denoted by π^c. The restrictions are $\rho^i_{y\pi} = 0$ and $\rho^i_{\pi y} = 0$ for all i.

D Data for the Public and Publicly Guaranteed (PPG) External Debt

This appendix shows the values of external debt that is either public or publicly guaranteed (PPG). Data is from the World Bank Database, at an annual frequency. Since the model is solved at the quarterly frequency, I compute the following

$$\left(\frac{b}{4 \times y}\right)^{\text{model}} = \left(\frac{b}{y}\right)^{\text{data}}$$

For each country in the sample, I compute the time series for the debt-to-output ratio. Then, I compute a summary of statistics for each country. I get the medians and then I the median between all the medians. This yields a target of 64.14%.

Country	min	mean	median	max	std
Argentina	49%	117%	99%	369%	82%
Brazil	14%	40%	39%	78%	19%
Chile*	-	-	-	-	-
Colombia	48%	74%	69%	103%	19%
Costa Rica	37%	69%	73%	94%	19%
Mexico	43%	65%	59%	100%	19%
Bulgaria	32%	104%	69%	273%	78%
Hungary*	-	-	-	-	-
Poland*	-	-	-	-	-
Romania	28%	63%	69%	81%	14%
Russia	30%	73%	50%	243%	52%
Turkey	44%	63%	54%	106%	20%
South Africa	21%	47%	37%	105%	26%
India	23%	39%	27%	81%	20%
Indonesia	48%	105%	83%	283%	58%

Table 9: No data on Debt available for Chile, Hungary, and Poland in the World Bank's database.

E Figures for the Baseline Case

The following figures show several objects of interest for the simple model under the baseline parametrization. Because there are three states, I plot two 2D figures for each object. First, I plot lines for three output levels, forcing the cyclical US dollar inflation to be exactly at its mean $\pi = \pi_m = 0$. Then, I do the opposite: I force the output to be at its mean $y = y_m = 1$ and plot three inflation levels. These three levels for each variable are given by 2 (two) unconditional standard deviations above and below the unconditional mean and its means for inflation and output.

Below there is a comprehensive list of figures.

Figure 6 — Value Function

Figure 7 — Price Schedule

Figure 8 — Price Schedule — Heatmap

Figure 9 — Borrowing Policy

Figure 10 — Revenue from Borrowing — Laffer Curve

Figure 11 — Consumption Policy

Figure 12 — Default Decision

Figure 13 — Repayment and Default Sets

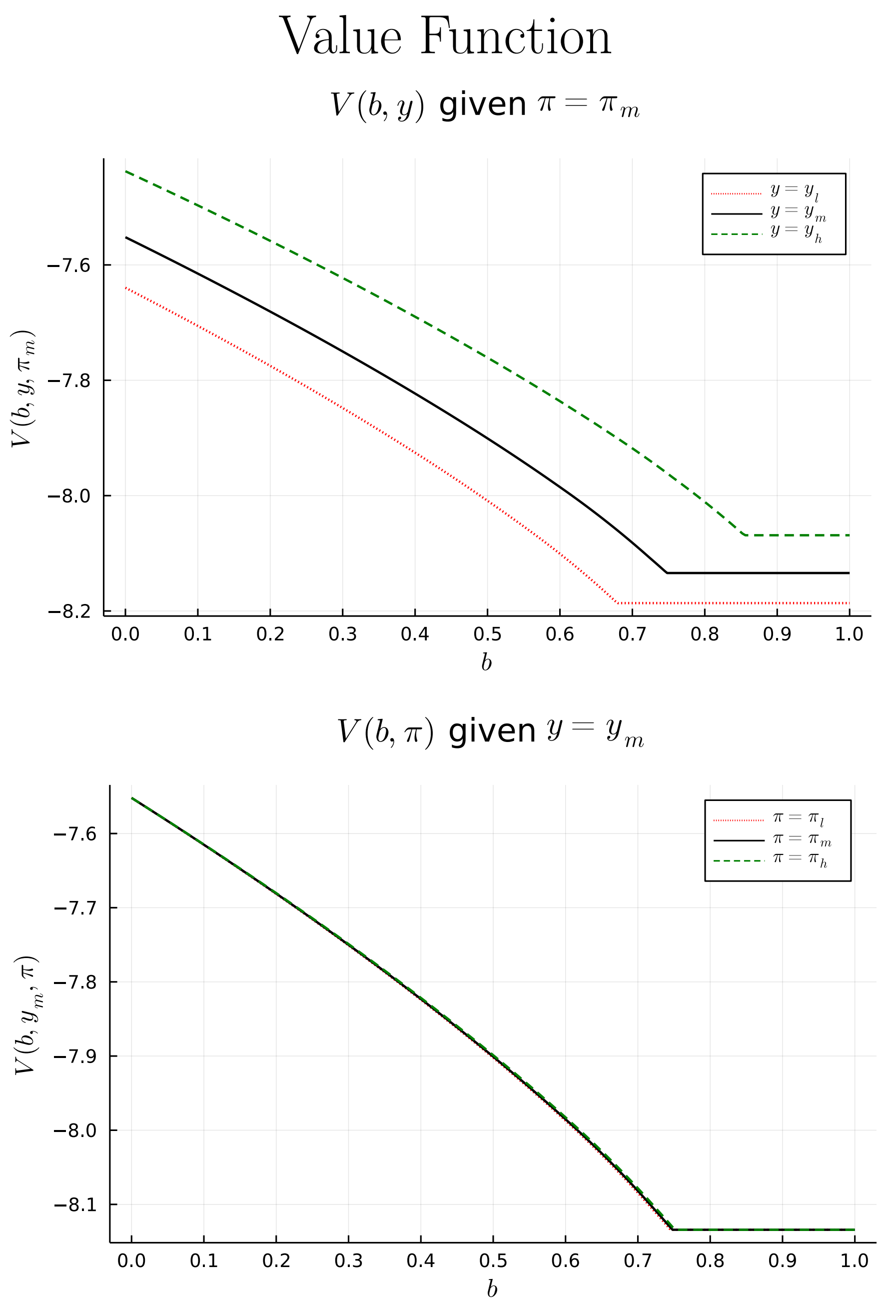


Figure 6: The top figure shows the Value Function for three levels of output, fixing inflation is at its mean, $\pi = \pi_m = 0$. The bottom figure shows the Value Function for three inflation levels, fixing output at its means, $y = y_m = 1$. These output levels are two unconditional standard deviations above the mean, the mean, and two unconditional standard deviations below the mean for output in the top figure and US dollar inflation in the figure at the bottom. By construction, the solid black line is the same in both figures. Each plot's flat portions (bottom right) highlight the default region, where the value does not depend on the debt level b. Value is weakly decreasing in b. The higher the output, the higher the Value. The higher the realized inflation, the higher the Value. The difference between inflation levels becomes more important as the debt level increases.

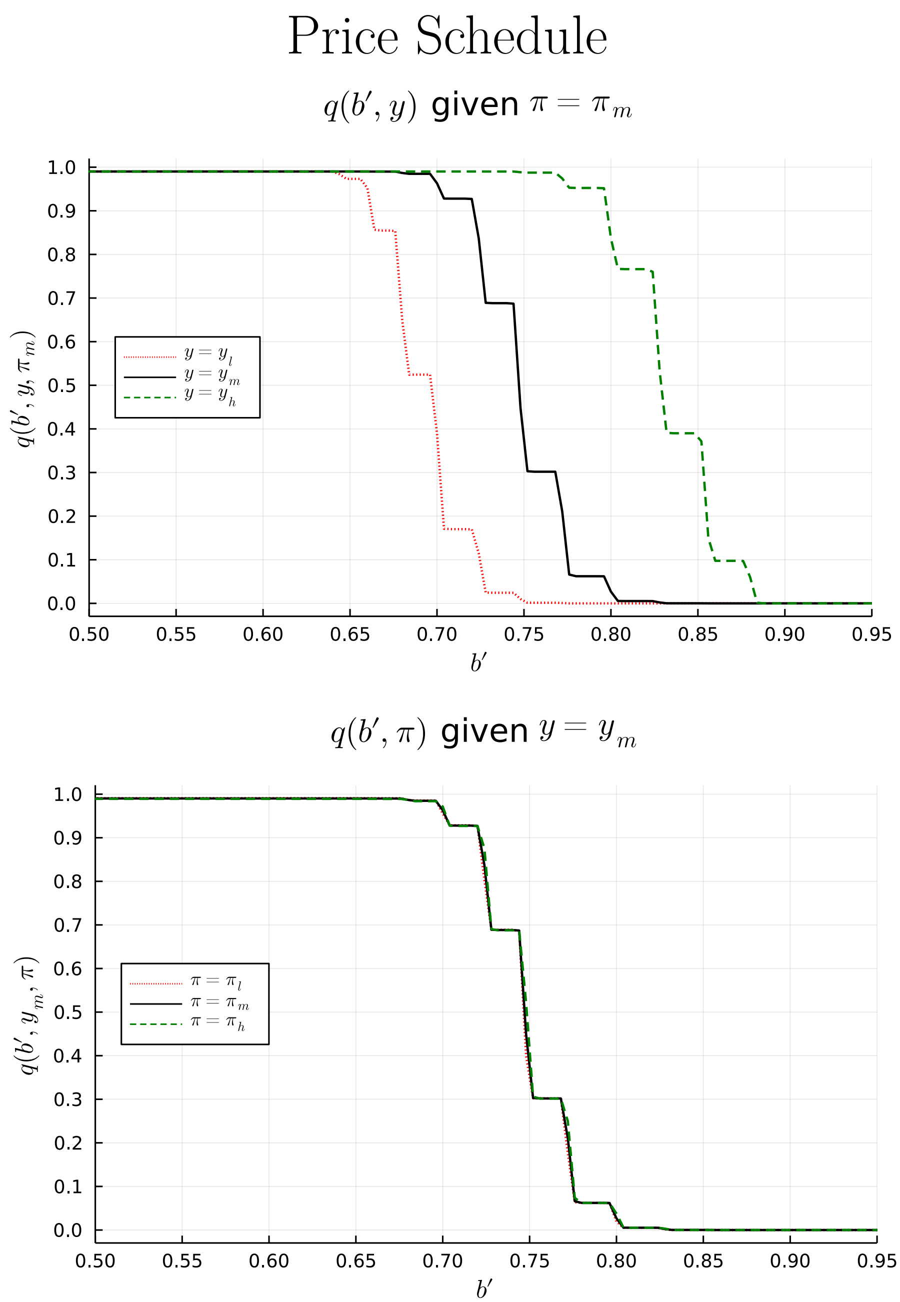


Figure 7: The top figure shows the Price Schedule for three levels of output, fixing inflation is at its mean, $\pi = \pi_m = 0$. The bottom figure shows the Price Schedule for three inflation levels, fixing output at its means, $y = y_m = 1$. These output levels are two unconditional standard deviations above the mean, the mean, and two unconditional standard deviations below the mean for output in the top figure and US dollar inflation in the figure at the bottom. By construction, the solid black line is the same in both figures. Each plot's flat portions (top left) highlight the region of b such that the bonds are risk-free regions. The bond price tends to decline weakly in the amount of borrowing b'. The higher the output, the better the price schedule. Since inflation is largely uninformative about future inflation and is completely disconnected from the output in the baseline, the price schedule is virtually independent of the current inflation prospects.

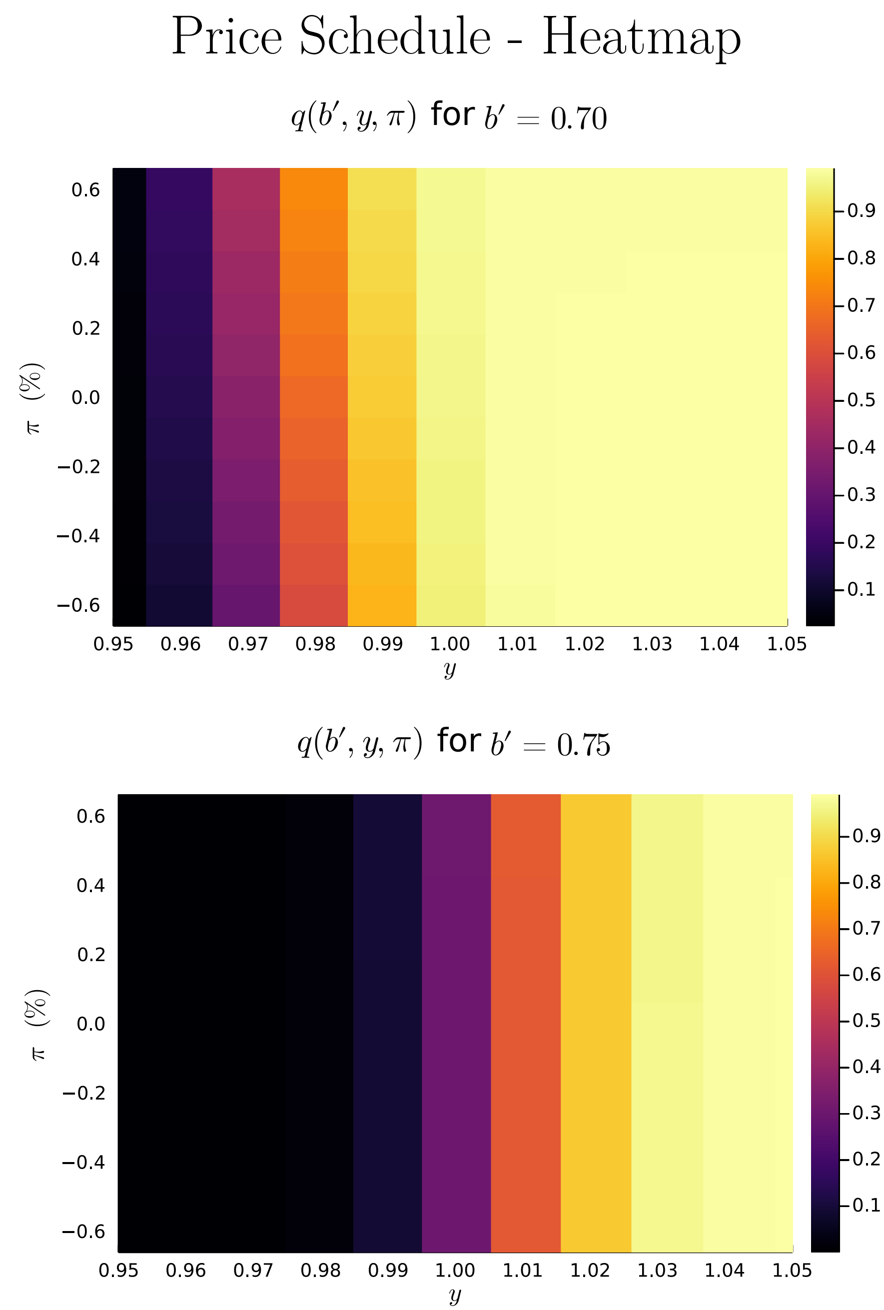


Figure 8: The top figure shows the Price Schedule fixing $b' = 0.70$ for across the grids for y and π. The top figure shows the Price Schedule fixing $b' = 0.75$ across the grids for y and π. One can see that the main driver for the price is the output level rather than the inflation level. This is the case because innovations to inflation are around five times less volatile than innovations to output. In addition, the persistence of output is around four times higher than the persistence of inflation, rendering current inflation poorly informative regarding inflation in the future.

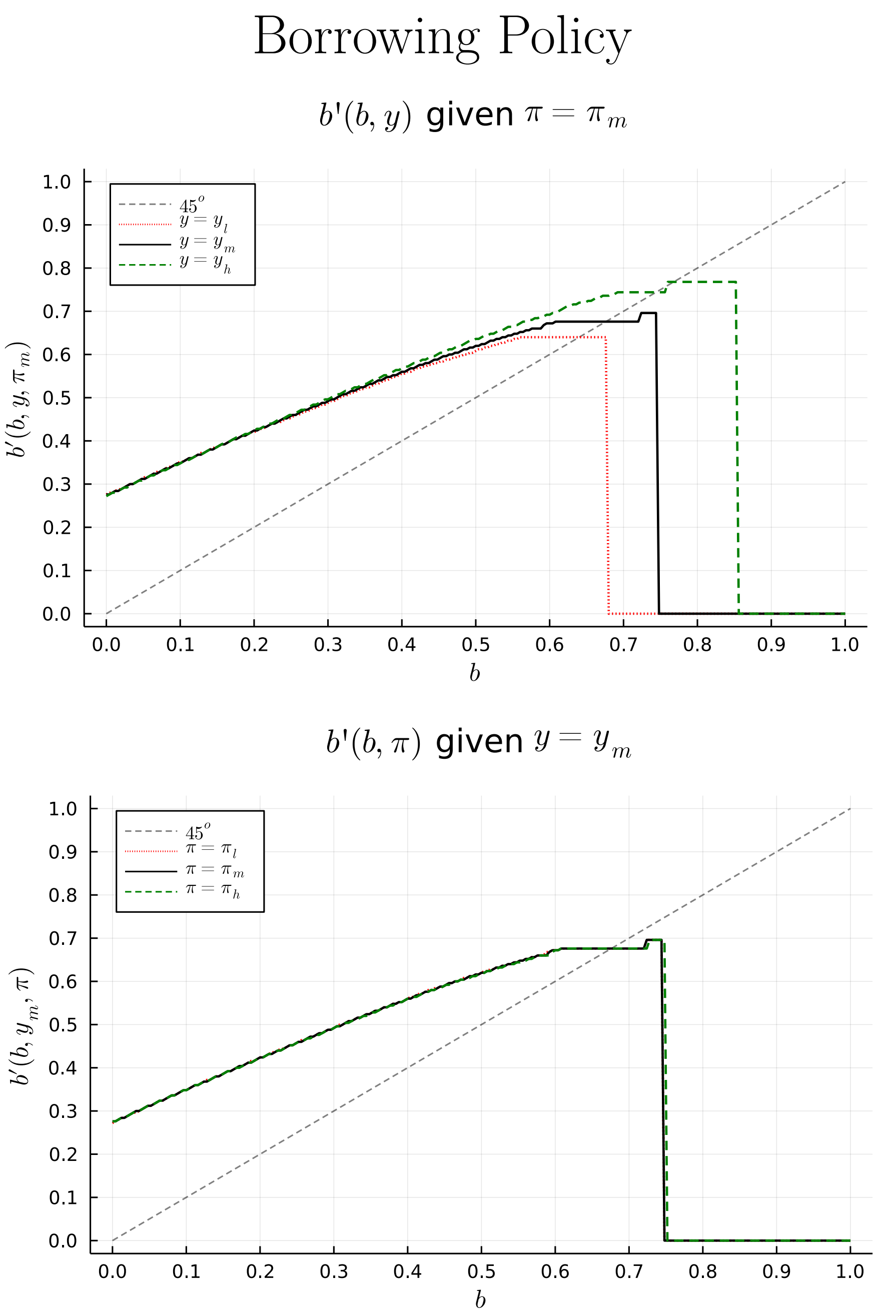


Figure 9: The top figure shows the Borrowing Policy for three levels of output, fixing inflation is at its mean, $\pi = \pi_m = 0$. The bottom figure shows the Borrowing Policy for three inflation levels, fixing output at its means, $y = y_m = 1$. These output levels are two unconditional standard deviations above the mean, the mean, and two unconditional standard deviations below the mean for output in the top figure and US dollar inflation in the figure at the bottom. By construction, the solid black line is the same in both figures. The plot shows the 45^o for comparison. The flat portions (bottom right) of each plot highlight the region of b such that the country would be in default, so $b' = 0$ is the optimal behavior. The main driver of borrowing is the output, as emphasized in the other two pictures.

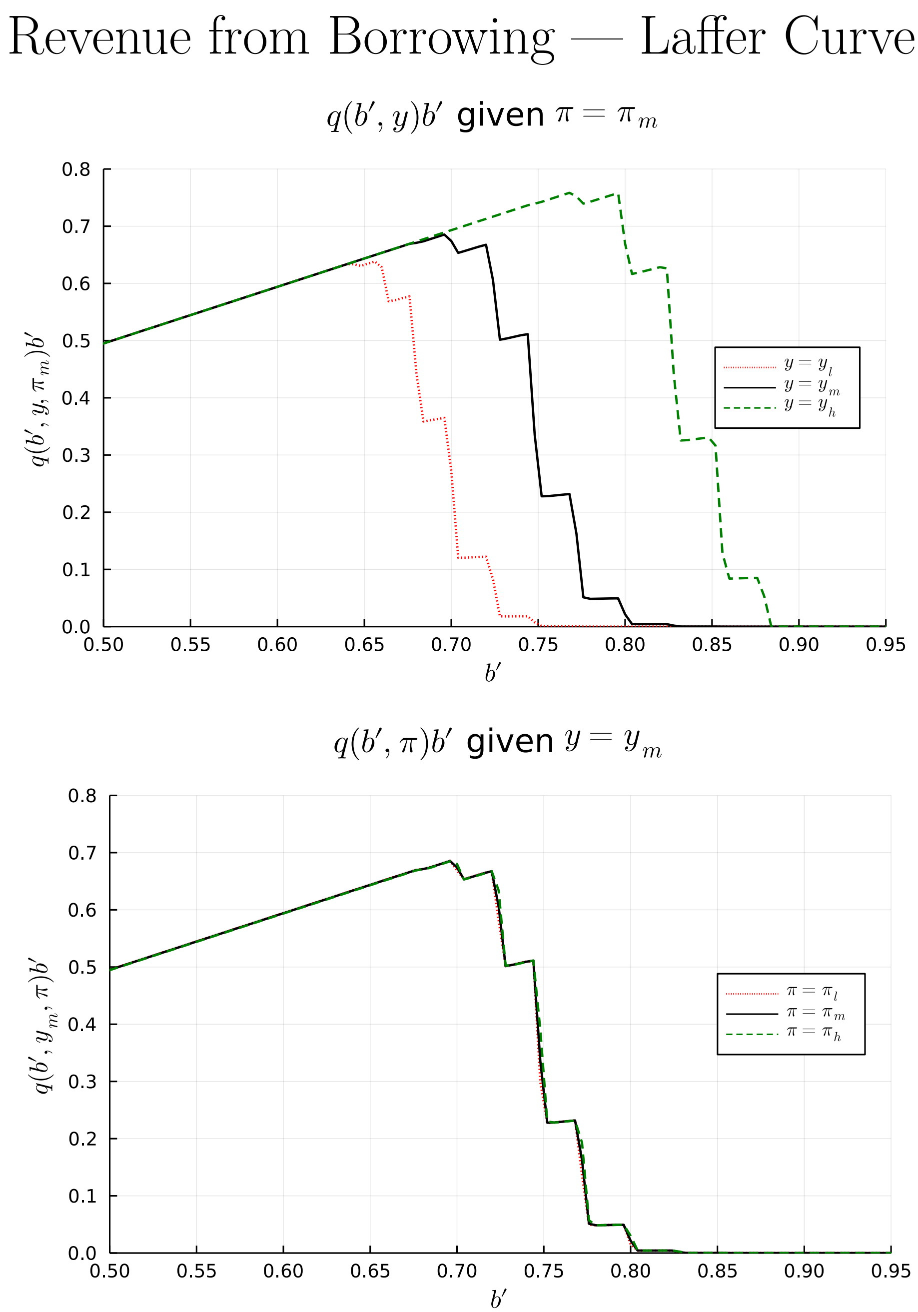


Figure 10: The top figure shows the Revenue from Borrowing for three levels of output, fixing inflation is at its mean, $\pi = \pi_m = 0$. The bottom figure shows the Revenue from Borrowing for three inflation levels, fixing output at its means, $y = y_m = 1$. These output levels are two unconditional standard deviations above the mean, the mean, and two unconditional standard deviations below the mean for output in the top figure and US dollar inflation in the figure at the bottom. By construction, the solid black line is the same in both figures. Here, the government would never choose b' to the right of the highest peak of the Laffer Curve (around 70% for mean output and mean inflation, represented by the solid black line) since reducing the indebtedness for tomorrow (moving to the left) would increase both the continuation value and the flow utility of the current period.

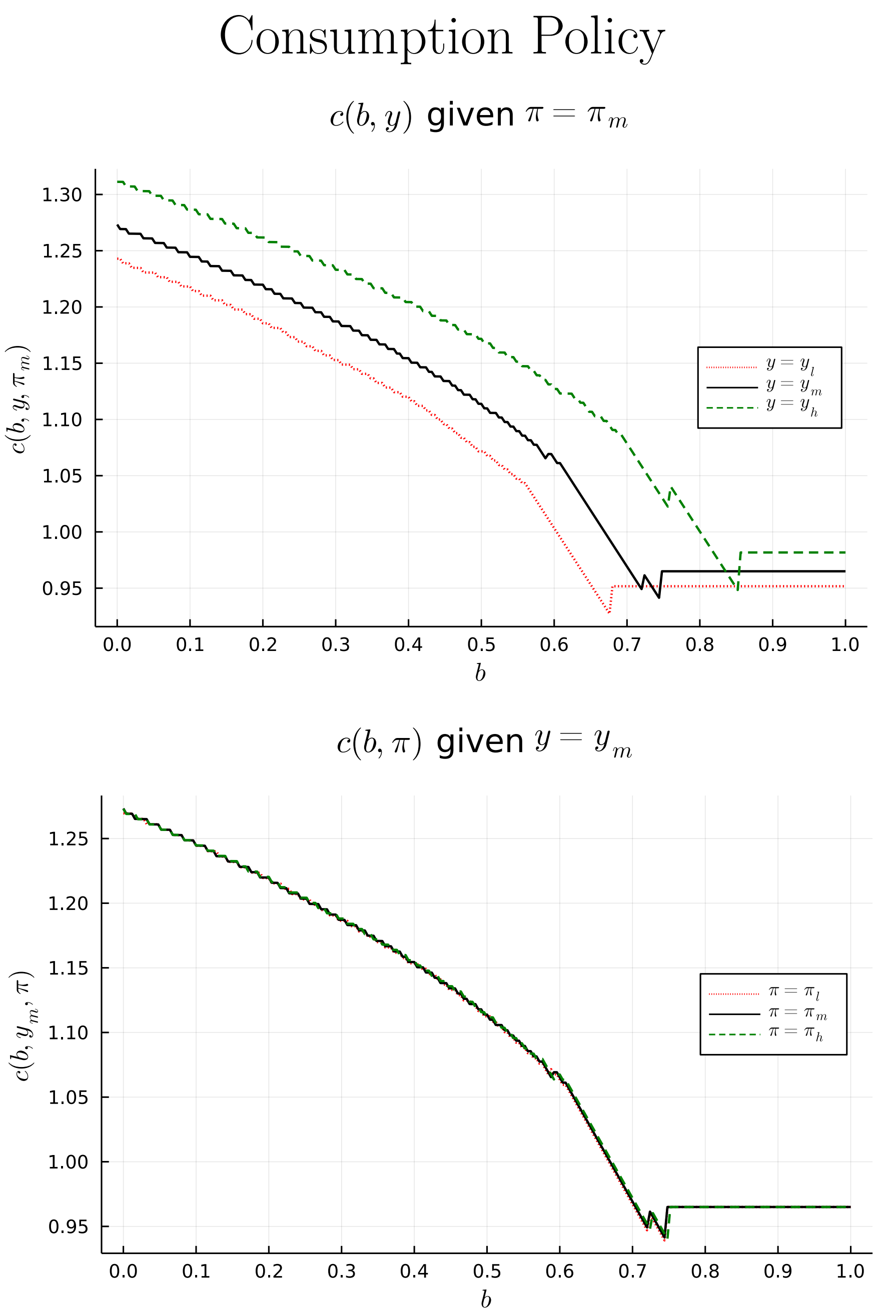


Figure 11: The top figure shows the Consumption Policy for three levels of output, fixing inflation is at its mean, $\pi = \pi_m = 0$. The bottom figure shows the Consumption Policy for three inflation levels, fixing output at its means, $y = y_m = 1$. These output levels are two unconditional standard deviations above the mean, the mean, and two unconditional standard deviations below the mean for output in the top figure and US dollar inflation in the figure at the bottom. High output and high realized inflation are associated with high consumption. The flat region (right bottom) represents the default region, where consumption depends on output (not on inflation) and is independent of the debt level. Consumption depends on income since $c = y - h(y)$, but the RHS of this equation is independent of the inflation level.

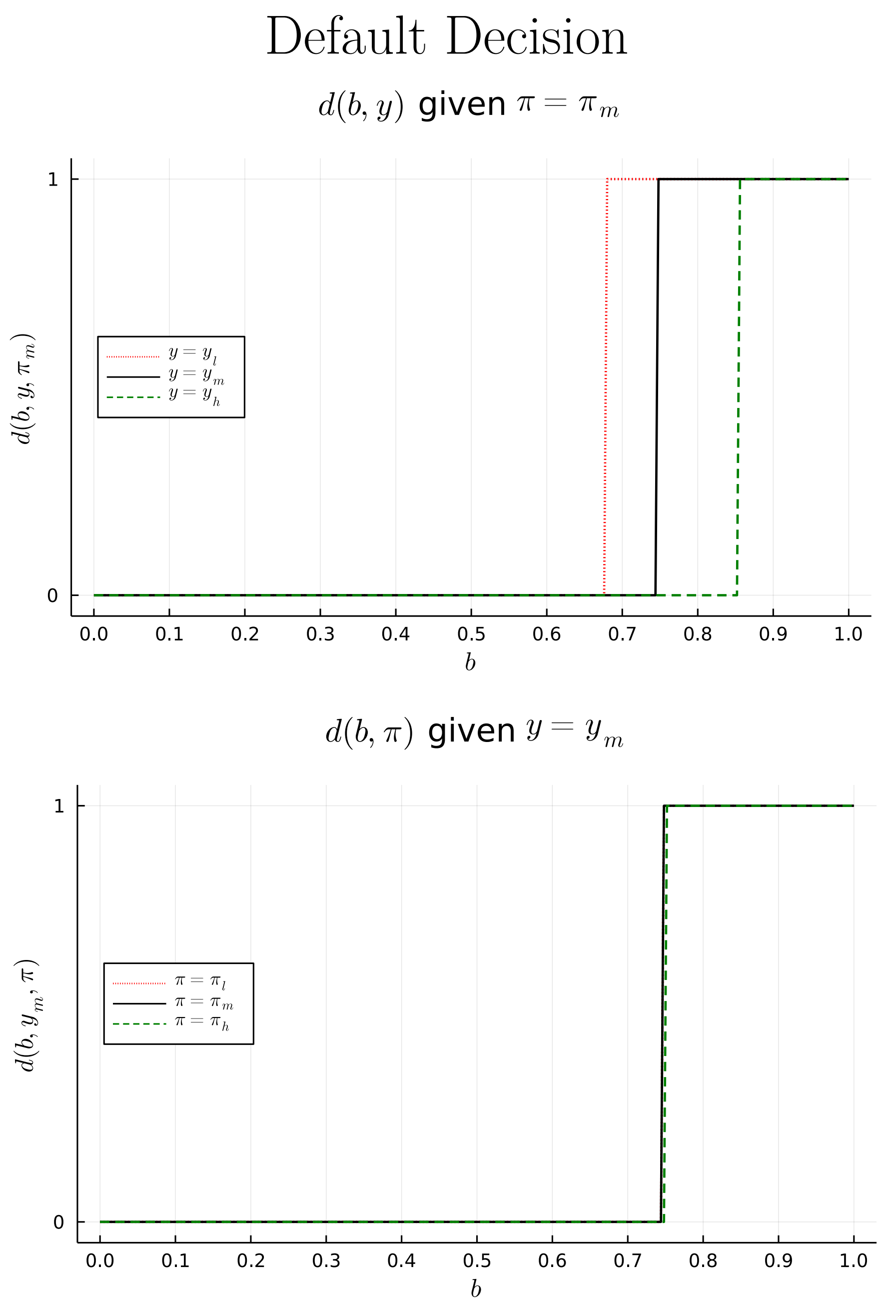


Figure 12: The top figure shows the Default Decision Policy for three levels of output, fixing inflation is at its mean, $\pi = \pi_m = 0$. The bottom figure shows the Default Decision for three inflation levels, fixing output at its means, $y = y_m = 1$. These output levels are two unconditional standard deviations above the mean, the mean, and two unconditional standard deviations below the mean for output in the top figure and US dollar inflation in the figure at the bottom. The government default with high output only with high debt. The same logic holds for inflation. A high realized inflation lowers the debt repayment burden, and the government decides optimally to default only with higher debt, compared to mean inflation. Again, the main driver of default is the output level.

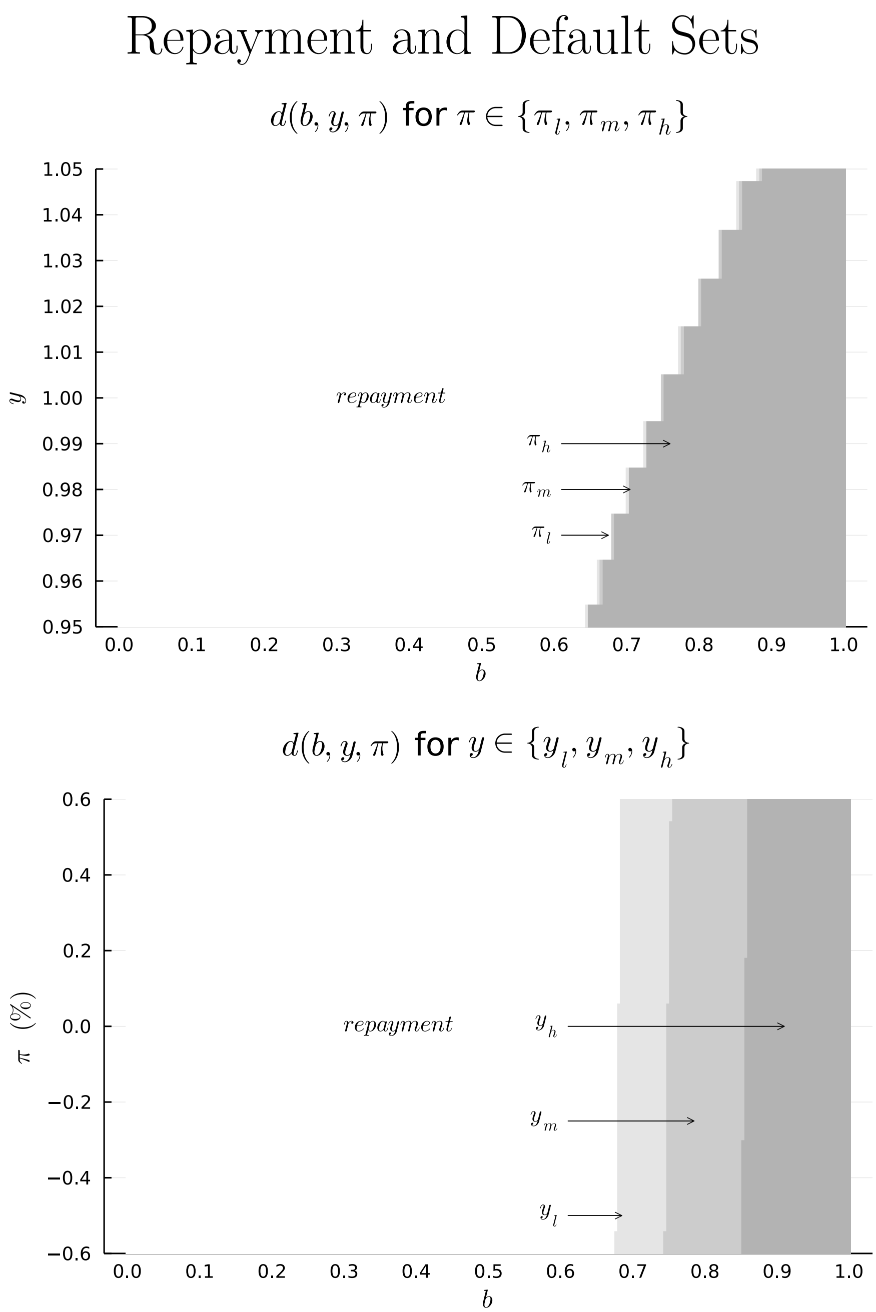


Figure 13: The top figure shows the Default Sets along the grids for debt (horizontal) and output (vertical), fixing three levels of inflation. The bottom figure shows the Repayment and Default Sets along the grids for debt (horizontal) and inflation (vertical), fixing three output levels. These output levels are two unconditional standard deviations above the mean, the mean, and two unconditional standard deviations below the mean for output in the bottom figure and for US dollar inflation in the figure at the top. The default sets are nearly invariant to different inflation levels (but not exactly), while they change drastically with the output level realized. This can be verified by the slope of the boundary in the top figure and the horizontal space between the boundary in the bottom figure.

F Scatter Plots for all EMs

In this appendix, I show the scatter relating the cyclical components of GDP for each EM in my sample and the US dollar inflation, as in figure 1.

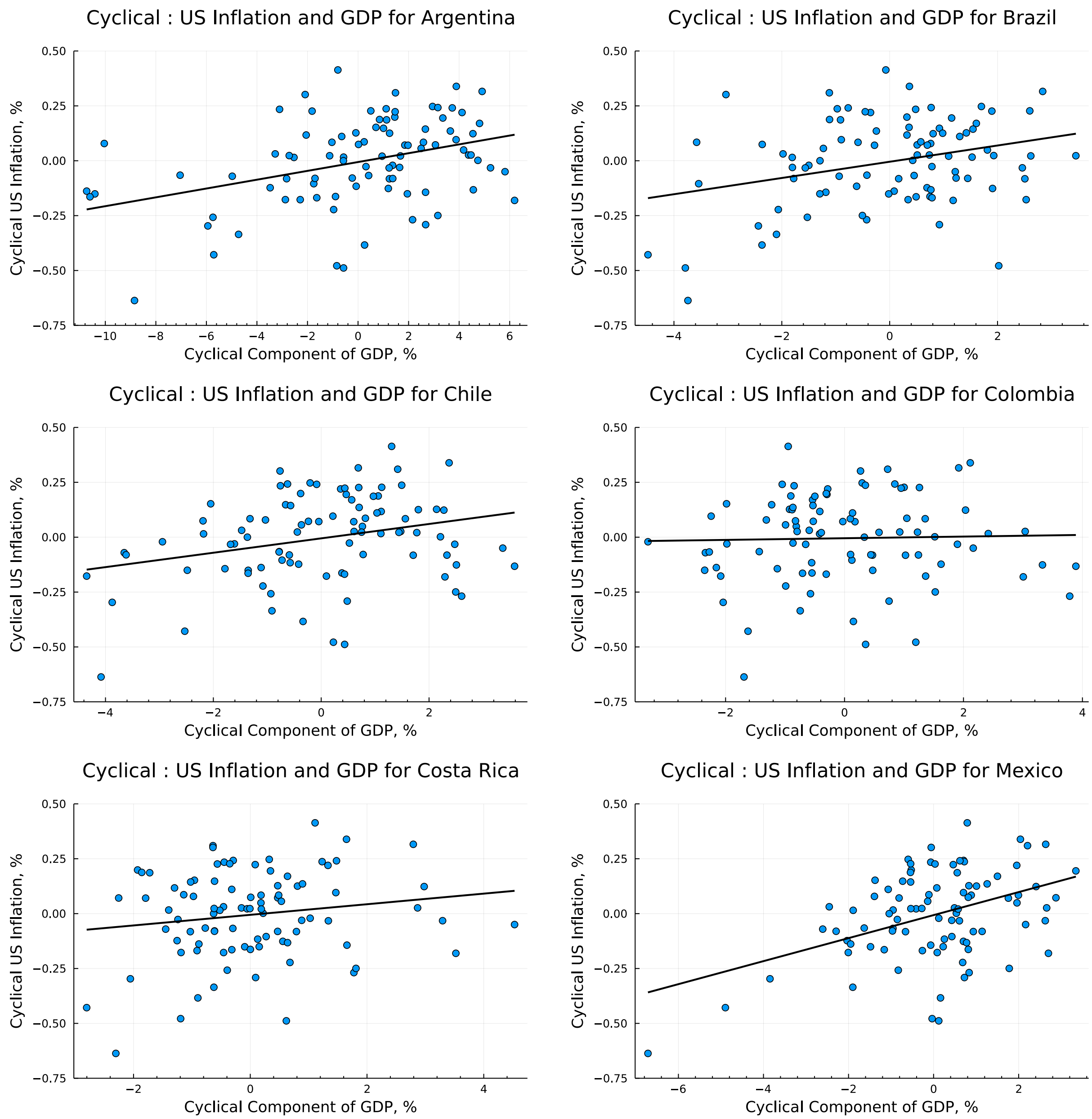


Figure 14: Cyclical US Inflation and GDP for countries in Latin America. The solid black line corresponds to the best-fit slope. Each dot in the plot corresponds to one-quarter of the data.

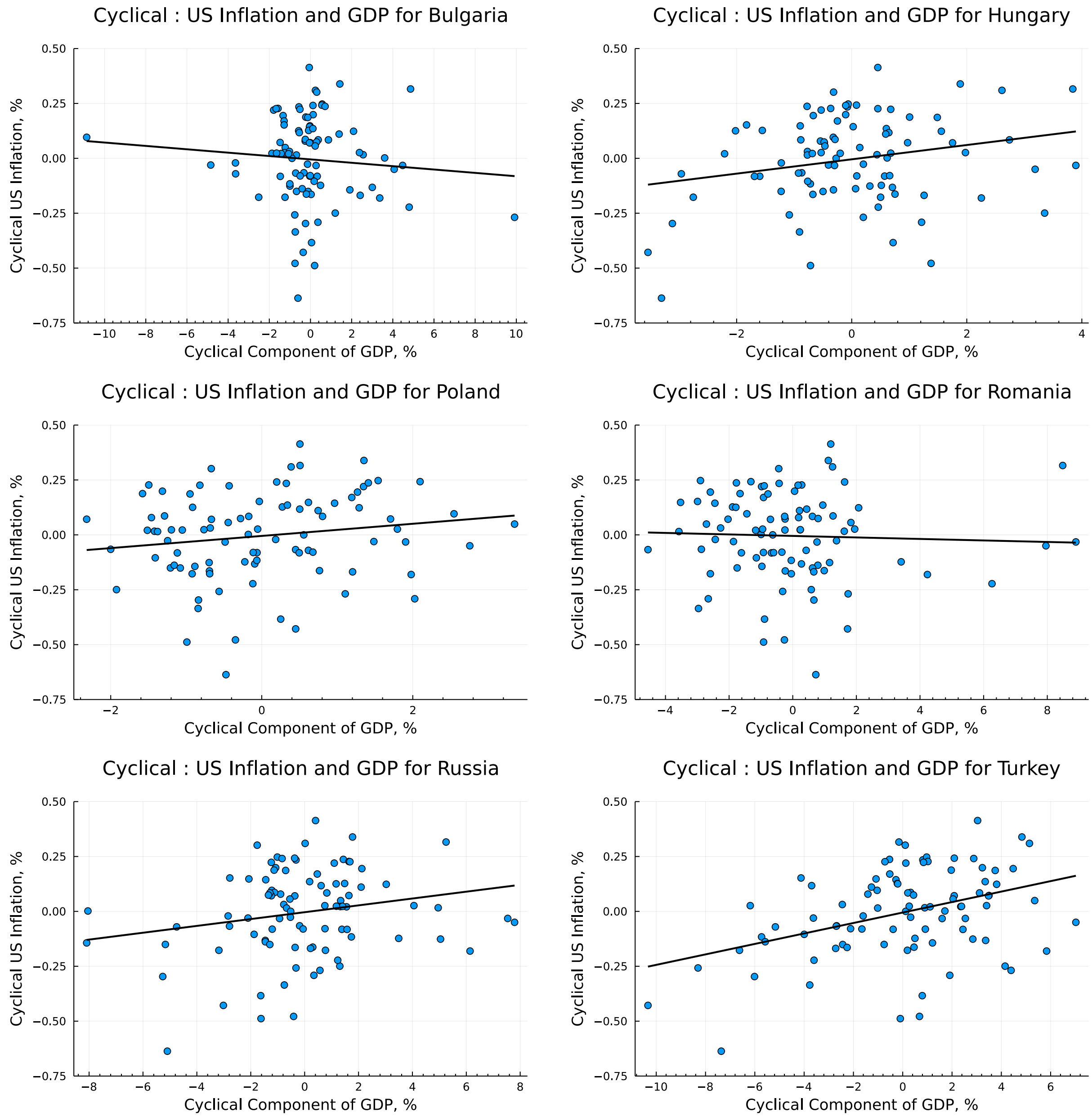


Figure 15: Cyclical US Inflation and GDP for countries in Europe. The solid black line corresponds to the best-fit slope. Each dot in the plot corresponds to one-quarter of the data.

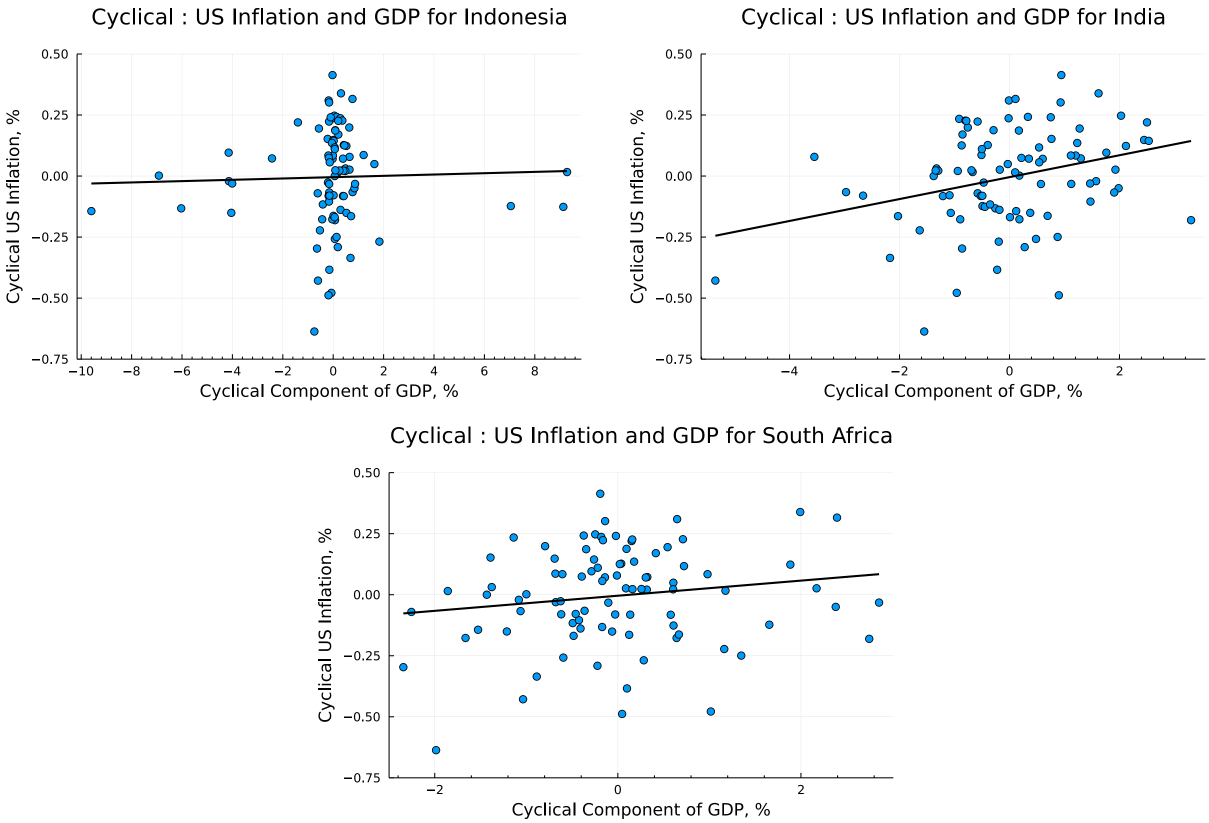


Figure 16: Cyclical US Inflation and GDP for countries in Asia and Africa. The solid black line corresponds to the best-fit slope. Each dot in the plot corresponds to one-quarter of the data.